

COUPLING OF DG METHODS WITH ONE INTEGRAL EQUATION

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Key words: *Discontinuous Galerkin methods, Boundary Elements, couple problems.*

Boundary Integral Equation methods can be used as precise (while expensive) methods to create and discretize exact absorbing boundary conditions for different problems. Reversely, Discontinuous Galerkin methods can be used to model the interior of non-homogeneous scatterers when hit by linear waves that have been modelled and discretized with BEM. The simplest form of coupling volume methods (DG, FEM, FVM) with boundary methods (generally Galerkin BEM) uses one integral identity to create a non-local boundary condition. Only quite recently the mathematical justification of this coupling has been understood for the simple FEM-BEM coupling on polygonal boundaries, although there are still some open questions.

In this talk I will present two forms of coupling DG with BEM. For primal formulation style DG (all the many variants of IP fit in this category), the method was proposed and experimented without analysis in [3]. The analysis was developed in [1], and required the development of some new tools for scaling arguments. In the case of the HDG method, it is interesting to see that the method interacts with the BEM as a mixed method, and thus has to use a different boundary integral equation, similar to what happens to RT schemes. A recent paper [2] shows how to analyze this coupling (improving at the same time some previous estimates of RT-BEM coupling).

I will comment on these two papers and will show some experiments of HDG-BEM, showing how the analysis is still not able to justify everything that can be computed.

REFERENCES

- [1] N. Heuer and F.J. Sayas. Analysis of a non-symmetric coupling of Interior Penalty DG and BEM. *Math. Comp.*, to appear.
- [2] Z. Fu, N. Heuer, and F..J. Sayas. Coupling of HDG with a double-layer potential BEM. Submitted

- [3] G. Of, G. Rodin, O. Steinbach, and M. Taus. Coupling of Discontinuous Galerkin Finite Element and Boundary Element Methods. *SIAM J. Sci. Comp.* 34(3), A1659A1677, 2012.