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Comparison of various approaches for modelling interaction of dispersive media and electromagnetic waves.

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In this paper various approaches for calculation transient response of systems containing frequency depended materials excited by an electromagnetic wave are compared. As an example, implementation of Debye media. The frequency dependence of Debye media can be expressed as a sum of single pole systems on s plane. Efficiency and accuracy of different methodologies is compared: inverse Fourier transform of a multiple frequency domain simulations and direct modification of time domain marching schemes. The latter is presented in a formalism of digital filters.

Introduction

Dispersive materials play an important role in modern computational electromagnetics and electromagnetic compatibility. For instance biological tissues and a large variety of organic liquids satisfy Debye relaxation model [1]. Capabilities of modern computers allow simulating complex structures, including interaction of human body and electromagnetic waves, microwave tomography etc. Rigorous analysis of problems involving living tissues requires appropriate handling of dispersive materials. Application of Debye model can be found in [2], however the results are restricted to finite difference scheme. An attempt to solve the problem in finite element time domain method have been presented in [5]. This work partially extends that work, and special attention is paid to differences between classical 2nd order time marching scheme used in finite difference simulations and Newmark scheme, used in finite element method. Another problem discussed in the paper is an inverse Fourier transformation of a multiple frequency domain simulations.

Initial-boundary value problem is described by Maxwell equations or wave equation with appropriate boundary conditions. The dispersive material can be specified by a first order Debye dispersion

$$\mathbf{D}(\omega) = \left(\varepsilon_\infty + \sum_i \frac{\Delta\varepsilon}{1 + j\omega\tau_i} \right) \mathbf{E}(\omega) \qquad (1)$$

where **D** stands for the electric displacement vector and **E** denotes the electric field. Value ε_∞ is the hypothetical permittivity for an infinite frequency. $\Delta\varepsilon$ is related to a low frequency permittivity and τ is considered as relaxation time. In fact, the core of the problem is handling

$$\frac{\Delta\varepsilon}{1 + j\omega\tau} \qquad (2)$$

Thus in the further part a single pole (i=1) problem is considered.

Different schemes: In the paper we discuss two basis approaches: based on Fourier transformation [3] and digital filtering. Since the dispersive relation in frequency domain is straightforward, in fact no special handling of dispersive material is needed. However, in order to obtain wideband or transient response one need to solve a number of linear problems, one problem per frequency point. Solving a number of linear problems may seem an inefficient approach, however taking into account the following aspects puts a problem in a better context. Namely, in modern multicore, multiprocessor architectures this problem is perfectly scalable. Secondly, one can assume that the solution does not significantly differ between two neighboring frequency points. This fact can be utilized when iterative algorithms for linear problems are used, which significantly reduces computational effort. Note, that in the case of the time domain finite element method, one linear problem per iteration is solved. Thus the difference in efficiency of direct and indirect formulation is not significant. The indirect approach, based on inverse Fourier transform is beneficial for analysis media of complicated frequency characteristics, especially those described by a large number of Debye poles (1).

Among purely time domain algorithms, the dispersive relation is held as a digital filter. It means that the sample of the quantity of interest is a convolution of media's pulse response and the incident field. For the Debye media the dispersive part of (1), corresponds to exponentially decaying pulse response. Such a response can be effectively modeled with a single pole digital filter of a form (keeping notation of (1)):

$$D[n \Delta t]=a D[(n-1)\Delta t] + E[n \Delta t] \qquad (3)$$

However, the one-to-one transformation of a pulse response to discrete time systems results in errors, related to nonlinear mapping between frequencies in continuous time and discrete time systems.

In [5] authors employed so called bilinear mapping, which resulted with an accurate, but less computationally effective algorithm. The main drawback of the bilinear mapping is need of keeping in memory an additional state variable, compared to (3). In the work the authors exploit the difference between linear and bilinear mapping for 2^{nd} order and Newmark time marching scheme.

Summary:

The accuracy and efficiency of handling dispersive materials in time domain algorithms is shown. Approach based on direct and bilinear transform of a pulse response of Debye medium is compared. Also indirect approach, namely obtaining the time depended signal as an inverse Fourier transform of results of multiple frequency domain simulations is discussed.

REFERENCES

[1] LAZEBNIK, M., OKONIEWSKI, M., BOOSKE, H., HAGNESS, S. C. Highly accurate debye models for normal and malignant breast tissue dielectric properties at microwave frequencies. MWCL, vol. 17, no. 12, p. 822-825, 2007.

[2] MROZOWSKI, M., OKONIEWSKI, M., STUCHLY, M. A. Simple treatment of multi-term dispersion in FDTD. MWGL, vol. 7, no. 5, p. 121-123, 1995.

[3] A. Taflove, S.C. Hagness and S.D. Gedney, Computational Electrodynamics: *Finite-Difference Time-Domain Methods,* 6^{th} Edition, Elsevier, 2005.

[4] J. Jin, D.J. Riley,*Finite Element Analysis of Antennas and Arrays*, 2009

[5] M. Wiktor, J. Ciganek, *Digital Filter Based Analysis of Debye Dielectric Media in Time Domain,* 2012