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# ASYMPTOTIC-PRESERVING SEMI-LAGRANGIAN DISCONTINUOUS GALERKIN SCHEMES FOR A CLASS OF RELAXATION SYSTEMS

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We consider in this work the class of singularly perturbed hyperbolic balance laws of the form

$$q_{,t} + \frac{1}{\varepsilon} A^1 q_{,x^1} + \frac{1}{\varepsilon} A^2 q_{,x^2} = \frac{1}{\varepsilon^2} B q, \tag{1}$$

where $q(t, \mathbf{x}) : \mathbb{R}^+ \times \mathbb{R}^2 \to \mathbb{R}^M$ and $A^1, A^2, B \in \mathbb{R}^{M\times M}$. We restrict ourselves to systems of the form (1) that admit a diffusive limit when $\varepsilon \to 0^+$. Such systems arise naturally in radiative transport applications such as neutron transport if one starts with a fully kinetic Boltzmann description and expands the distribution function in spherical harmonics. The truncation of this spherical harmonics expansion is often referred to as the $P_n$ approximation in radiative transport.

One key difficulty in solving systems of the form (1) is that standard numerical schemes have time-step restrictions that are proportional to $\varepsilon$; and therefore, these methods become impractical in the diffusive limit. Several approaches have been proposed in the literature that overcome this difficulty (e.g., Jin [1], Jin et al. [2], and McClarren et al. [3]). These approaches are based on splitting the equation into stiff and non-stiff pieces and using appropriate semi-implicit time-stepping methods to achieve an overall scheme that has a time-step restriction that remains bounded away from zero as $\varepsilon \to 0^+$.

In this work we employ a different strategy in order to achieve asymptotic-preservation. We develop an asymptotic-preserving scheme using a high-order discontinuous semi-Lagrangian Galerkin scheme. This scheme is based on ideas from the semi-Lagrangian Vlasov-Poisson solver of Rossmanith and Seal [4]. Novel modifications are introduced to generalize the scalar scheme of [4] to system of conservation laws of the form (1).

We demonstrate analytically and numerically that the proposed scheme is asymptotic-preserving in the diffusive limit. Several numerical test cases are used to validate the proposed scheme.

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