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ON THE PATH DEPENDENCE OF COHESIVE ZONE ELEMENTS UNDER MIXED-MODE FRACTURE

Bent F. Sørensen* and Stergios Goutianos

Department of Wind Energy, Section of Composites and Materials Mechanics, Technical University of Denmark, Risø Campus, Building 228, Frederiksborgvej 399, DK-4000 Roskilde, Denmark
Email address: bsqr@dtu.dk, gout@dtu.dk; URL: www.vindenergi.dtu.dk

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The concept of modelling fracture with cohesive laws (describing the fracture process through a traction–separation relationship) was introduced by Dugdale [1] and Barenblatt [2]. Since Needleman [3] introduced a mode I cohesive law model in a continuum mechanics finite element model, cohesive zone models have been used extensively to describe failure in a wide variety of materials and interfacial systems, see for example [4,5].

In many material systems, mixed mode fracture is observed along weak planes e.g. along interfaces in layered structures (composites) or adhesive joints. Under mixed mode fracture, the fracture process zone is subjected to both normal (δ_n) and tangential separations (δ_t). A detailed list of different types of mixed-mode cohesive (or traction-separation) laws can be found in [6] based on if the fracture energy is dependent or independent of the crack opening history.

When the fracture mechanism involves history-dependent phenomena such as plasticity or friction, then path dependent cohesive laws should be used. In all other cases, it is argued that path independent cohesive laws (derived from a potential function) should be preferred. This is because there are no published experimental data that show if real cohesive laws are opening path dependent or not. Using a cohesive law that is path dependent has a number of consequences / undesirable features such as a) there is no guarantee that the correct mixed mode fracture energy will be obtained at a given phase angle ($\phi = \tan^{-1}(\delta_t / \delta_n)$), b) the work of separation for a given phase angle cannot be calculated since it depends on the history of normal and tangential separation of each material point in the fracture process zone, and c) energy can be generated under cyclic loading [6].

As it is mentioned above, when the cohesive law are derived from a potential function, then path independence is satisfied. In many cases, however, mixed mode cohesive laws are constructed knowing the mode I and mode II cohesive laws and using interaction criteria for mixed mode fracture [6,7]. Such cohesive laws are implemented in commercial finite element packages. In these cohesive laws, the normal (T_n) and shear traction (T_t) vectors are aligned with the normal and tangential openings, respectively, and as a result they are called truss-like mixed mode cohesive laws:

$$\frac{\delta_{\rm n}}{\delta_{\rm t}} = \frac{T_n}{T_t} \tag{1}$$

It will be shown theoretically and numerically that when Eq. 1 holds, the mixed mode cohesive laws are path independent only when the fracture resistance (J_R) is independent of the phase angle of openings (or the phase angle of tractions, $\psi = \tan^{-1}(T_t / T_n)$, since Eq. 1 implies that $\phi = \psi$):

$$\frac{\partial J_R}{\partial \phi} = 0 \tag{2}$$

Eq. 2 is seldom satisfied in practise as experimental studies have shown that under mixed mode, the fracture energy depends on the phase angle of openings e.g. [8]. Thus, truss-like mixed mode cohesive laws are unintentionally path dependent.

In a recent work [9], it was shown that when Eq. (1) holds, rotational equilibrium is satisfied in the cohesive elements. Therefore, it is not possible to satisfy at the same time both path independence and rotational equilibrium. It will be shown, that the error introduced by the lack of rotational equilibrium is important only for large rotations/displacements, and therefore, it is preferred to satisfy path independence.

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