HIGHER DEGREE IMMERSED FINITE ELEMENT METHODS FOR INTERFACE PROBLEMS

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Simulating a physical procedure over a domain consisting of different materials often leads to an interface problem consisting of partial differential equations with discontinuous coefficients representing differential material properties, boundary (and initial) conditions, and jump conditions required by pertinent physics across the material interfaces. The discontinuity in coefficients leads to less regularity in the exact solution which often prevents a traditional finite element (FE) method to perform satisfactorily unless each element in the mesh contains essentially only one material. Immersed finite element (IFE) methods are non-traditional FE methods that can utilize interface-independent meshes. Meshes used by IFE methods allow interface elements formed by multiple materials and IFE shape functions on interface elements are constructed according to interface location and jump conditions; therefore, IFE methods can solve interface problems more efficiently in many applications that prefer interface-independent meshes. Most published IFE methods use lower degree polynomials with optimal but limited approximation capability. Developing higher degree IFE spaces is necessary for solving interface problems more efficiently because of their higher order approximation capability and the fact that many modern techniques, such as mixed formulations and \textit{hp}-adaptive procedures, demand finite element spaces based on polynomials of different degrees. This presentation will discuss basic ideas for constructing higher degree IFE spaces for solving problems whose coefficients have jumps across general material interfaces. It will also discuss prototypical applications of higher degree IFE methods. Numerical examples will be provided to illustrate features of higher degree IFE methods in these applications.