11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
July 20–25, 2014, Barcelona, Spain

MODEL AND MESH ADAPTIVITY FOR FRICTIONAL CONTACT PROBLEMS

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Key words: *Frictional Contact Problems, Error Estimation, Model Adaptivity.*

Frictional contact problems play an important role in many production processes, where the use of complex frictional laws to ensure an accurate modelling leads to high computational costs. One approach to reduce the numerical effort is given by mesh adaptivity based on goal oriented a posteriori error estimation, which is discussed, for instance, in [1]. An advanced idea is now not only to adaptively modify the mesh but also the underlying models based on a posteriori error estimators. In this note, we shortly describe the approach in the case of Signorini's problem with friction in mixed form using the notation of [1]:

$$\begin{aligned} (\sigma(u),\varepsilon(v)) + <\lambda_n, v_n> + (\lambda_t, s v_t)_{0,\Gamma_C} &= <l, v>, \\ <\mu_n - \lambda_n, u_n - g> + (\mu_t - \lambda_t, s u_t)_{0,\Gamma_C} &\leq 0, \end{aligned}$$

for all $v \in V$, all $\mu_n \in \Lambda_n$ and all $\mu_t \in \Lambda_t$. Here, s specifies the reference friction model. This problem is discretized with a mixed finite element approach leading to a discrete solution $(u_h, \lambda_{n,H}, \lambda_{t,H})$, where the usual nodal low order finite element approach is used to discretize the displacement. For the discretization of the Lagrange multipliers piecewise constant basis functions on coarser meshes as in [1] or biorthogonal basis functions leading to Mortar methods, see, e.g., [2], are applied.

The first and essential step for model adaptivity is to specify an admissible and consistent model hierarchy. One example of such a model hierarchy for friction laws is given by the following models: frictionless contact, Tresca friction, Coulomb friction, friction model by Betten. We refer to [3, Section 4.2] to a detailed description of the single models. A simplified friction model s^m is locally composed by choosing one of the models out of the hierarchy. The corresponding discrete solution is given by $(u_h^m, \lambda_{n,H}^m, \lambda_{t,H}^m)$.

The aim is now to derive a posteriori error estimates for the error

$$J(u, \lambda_n, \lambda_t) - J(u_h^m, \lambda_{n,H}^m, \lambda_{t,H}^m),$$

where J is an user specified possibly nonlinear output functional. To this end, the contact conditions are formulated with the help of a nonlinear complementarity (NCP) function such that we arrive at a semilinear problem. Here, the NCP function is given by

$$D(u_h, \lambda_{t,H})(\mu_{t,H}) := \int_{\Gamma_C} \mu_{t,H} \left(\max\{s, \|\lambda_{t,H} + u_{t,h}\|\} - s \cdot (\lambda_{t,H} + u_{t,h}) \right) do$$

for the reference friction model s. For the model adaptive friction law s^m, it is given by D^m. The approach presented in [4] to derive a posteriori error estimates concerning the model and the discretization error is applied on the given problem formulation. However, we have to pay special attention to the remainder terms due to the nondifferentiability of the NCP function. At last, we obtain the model error estimator $\eta^m = D^m(u_h, \lambda_{t,H})(\xi_{t,H}) - D(u_h, \lambda_{t,H})(\xi_{t,H})$ and the usual discretization error estimator η^h. The function $\xi_{t,H}$ is the Lagrange multiplier concerning the frictional variable of the dual problem, which corresponds in this case to the last step of a primal dual active set method for solving the reduced problem. The error estimators η^m and η^h are localized and normalized to obtain model and refinement indicators, respectively. In the adaptive strategy, the values η^h and η^m are compared and using a balancing strategy, it is decided, whether the mesh, the modeling or both should be improved. Then, standard techniques are applied for model improvement respectively mesh refinement. The application of the presented algorithms on a production process is discussed in [5].

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