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Cross-sectional Analysis of Pre-Twisted Thick Beams using Variational Asymptotic Method

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An asymptotically-exact methodology is presented for obtaining the cross-sectional stiffness matrix of a pre-twisted thick beam made of transversely isotropic materials. The beam is modeled without assumptions from 3-D elasticity. The strain energy of the beam is computed using the constitutive law and the kinematical relations derived with the inclusion of geometrical nonlinearities and initial twist. Large displacements and rotations are allowed, but small strain is assumed. The Variational Asymptotic Method (VAM) is used to minimize the energy, thereby reducing the cross-section to a point on the reference line with appropriate properties, yielding a 1-D constitutive law. Closed-form expressions are derived for the 3-D warping and stress fields. The model is capable of predicting interlaminar and transverse shear stresses accurately up to first order.

VAM is a mathematical technique, which can rigorously split the 3-D analysis of anisotropic beams into two problems: a 2-D analysis over the beam cross-sectional domain, which provides a compact form of the properties of the cross-sections, and a nonlinear 1-D analysis of the beam[1]. In this method as applied herein, the 2-D analysis is performed asymptotically by taking advantage of a material small parameter, namely the maximum allowable strain (ϵ), and two geometric small parameters, namely the ratio of maximum cross-sectional dimension to beam deformation wavelength ($\delta_h = h/l$) and the product of maximum cross-sectional dimension with the maximum pre-twist per unit length ($\delta_R = h * k_1$). 3-D strain components are derived using kinematics and arranged as orders of the small parameters. Warping functions are obtained by the minimization of strain energy subject to certain set of constraints that renders the 1-D strain measures well-defined. The zeroth-order 3-D warping field thus yielded is then used to integrate the 3-D strain energy density over the cross-section, resulting in the 1-D strain energy density which in turn helps identify the corresponding cross-sectional stiffness matrix (S^0), as follows:

$$S^0 = \begin{bmatrix} BHK_0 & 0 & 0 & 0 \\ 0 & 0.33BH^3K_0 & 0 & 0 \\ 0 & 0 & (BH^3/12)K_0 & 0 \\ 0 & 0 & 0 & (B^3H/12)K_0 \end{bmatrix} \tag{1}$$

where $K_0 = \frac{C_{11}+(C_{13}^2 C_{22}-2C_{12}C_{13}C_{23}+C_{12}^2 C_{33})}{C_{23}^2-C_{22}C_{33}}$, B is the breadth and H is the height of the rectangular cross-section.

For an isotropic material, the zeroth-order solution is equivalent to the classical Euler-Bernoulli beam theory, as K_0 boils down to its Young's modulus (E). For finding the next approximation to the 1-D energy density, the previous classical approximation of the warping is perturbed. The 1-D transverse shear strain terms ($2\gamma_{12}$ and $2\gamma_{13}$) figure in the 3-D strains of first-order and the first-order warping component is computed by minimization of the asymptotically first order accurate strain energy[2].The resulting first-order cross-sectional stiffness matrix (S1) is a 6x6 matrix with transverse shear stress components, as in the case of the Timoshenko beam theory, which however is asymptotically incorrect.

$$S^1 = \begin{bmatrix} BHK_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & S_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & S_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.33BH^3K_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & (BH^3/12)K_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & (B^3H/12)K_1 \end{bmatrix} \tag{2}$$

where $K_1 = K_0$, $S_{22} = 2.133BHC_{66} + \frac{3.556H^3C_{66}^2}{BC_{55}} + \frac{0.8H^5C_{66}^3}{B^3C_{55}^2}$
and $S_{33} = 2.133BHC_{55} + \frac{0.8B^5C_{55}^3}{H^3C_{66}^2} + \frac{3.556B^3C_{55}^2}{HC_{66}}$

Hence the model yields an accurate semi-analytical solution for moderately thick beams with rectangular cross-sections. This model can be further expanded for an n-ply laminate, with different ply orientations, to explore the interlaminar stresses at various locations along the length of the beam. The results for a simple two-ply laminate [0/0] is given below:

$$\bar{S} = \begin{bmatrix} B(K[1]t_1 + K[2]t_2) & 0 & B(-K[1]t_1^2 + K[2]t_2^2) & 0 \\ 0 & 0.33B(C_{66}[1]t_1^3 + C_{66}[2]t_2^3) & 0 & 0 \\ B(-K[1]t_1^2 + K[2]t_2^2) & 0 & (13B/12)(K[1]t_1^3 + K[2]t_2^3) & 0 \\ 0 & 0 & 0 & (B^3/12)(K[1]t_1 + K[2]t_2) \end{bmatrix}$$

where $K[1/2] = \frac{C_{11}[1/2]+(C_{13}^2[1/2]C_{22}[1/2]-2C_{12}[1/2]C_{13}[1/2]C_{23}[1/2]+C_{12}^2[1/2]C_{33}[1/2])}{C_{23}^2[1/2]-C_{22}[1/2]C_{33}[1/2]}$

Helicopter rotors consisting of composite tapered flex beams are subjected to high cycles of fatigue and are prone to delamination at the ply drop-off regions. Such complicated structures can be analysed using this model and closed-form solutions can be obtained to have a better understanding of thick beams.

REFERENCES

[1] D. H. Hodges, *Nonlinear Composite Beam Theory*. Progress in Astronautics and Aeronautics, Vol. 213, 2006..

[2] M. V. Peereswara Rao, D. Harursampath and K. Renji, *Prediction of Interlaminar Stresses in Composite Honeycomb Sandwich Panels under Mechanical Loading using Variational Asymptotic Method*. Composite Structures, Vol. 94(8), pp. 2523-2537, 2012.