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SOLVING INTERFACE PROBLEMS BY THE REGULARIZED METHOD OF FUNDAMENTAL SOLUTIONS

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Interface problems play an important role in describing heat transfer in composite objects like in parts of electric and hybrid cars, seepage through inhomogeneous (and possibly anisotropic) media etc. We restrict ourselves to the elliptic problem

$$\mathrm{div}\,\sigma\,\mathrm{grad}\,u = 0 \tag{1}$$

defined in a domain Ω, where σ is a given, positive function, often assumed to be piecewise constant. That is, the original domain Ω is split into disjoint subdomains Ω_1, Ω_2, ..., Ω_M and σ is identically equal to the constant $\sigma_k > 0$ in Ω_k. Then (1) is split into a finite set of Laplace equations:

$$\Delta u_k = 0 \qquad \text{in } \Omega_k \tag{2}$$

supplied with some familiar boundary conditions along the boundary of the domain Ω and interface conditions along the non-empty interfaces $\Gamma_{kj} := \partial\Omega_k \cap \partial\Omega_j$:

$$u_k|_{\Gamma_{kj}} = u_j|_{\Gamma_{kj}}, \qquad \sigma_k \cdot \frac{\partial u_k}{\partial n_{kj}}|_{\Gamma_{kj}} = \sigma_j \cdot \frac{\partial u_j}{\partial n_{kj}}|_{\Gamma_{kj}}, \tag{3}$$

where n_{kj} is the normal unit vector along Γ_{kj} pointing from Ω_k to Ω_j.

Since the fundamental solution of (1) cannot be expressed in a convenient form, the traditional Method of Fundamental Solutions (MFS, see e.g. [1]) can be applied to the equations of (2) separately. In its original form, this requires sets of source points located outside of the corresponding subdomains. A more comfortable technique is when the source and the boundary collocation points are allowed to coincide. This requires some regularization technique to avoid the problem of singularity of the fundamental solution of the Laplacian. The simplest regularization is based on truncation; another possibility is to replace the fundamental solution of the Laplace operator with that of a singularly perturbed fourth-order operator $\Delta(I - \frac{1}{c^2}\Delta)$, where $c > 0$ is a predefined scaling constant and should be inversely proportional to the characteristic distance of the boundary collocation points; then the fundamental solution approximates the harmonic fundamental solution but it is continuous everywhere, see [2]. However, to properly handle also the singularities of the normal derivatives of the fundamental solution, special desingularization techniques are needed, see e.g. [2], [3] for details.

In this paper, the above mentioned regularization and desingularization techniques are applied to the interface problem (2)-(3). Due to the desingularization, the interface conditions are approximated by defining additional sources located along the interfaces. This results in an algebraic system which is much better conditioned than in the case of the traditional MFS. The approach works also in the case of 3D problems. Some numerical examples are also presented.

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