11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
July 20–25, 2014, Barcelona, Spain

NUMERICAL SIMULATION OF FREE SURFACE FLOWS, WITH MULTIPLE LIQUID PHASES

Alexandre Caboussat[1,*], N. James[2], S. Boyaval[3] and Marco Picasso[4]

[1] Geneva School of Business Administration, HES-SO, Route de Drize 7, 1227 Carouge, Switzerland, alexandre.caboussat@hesge.ch, http://campus.hesge.ch/caboussata
[2] Université de Poitiers, Lab. de Mathématiques, 86962 Futuroscope Chasseneuil Cedex, France, Nicolas.James@math.univ-poitiers.fr, http://www-math.univ-poitiers.fr/ njames/
[3] Université Paris Est, Laboratoire d'hydraulique Saint-Venant 78401 Chatou Cedex, France, http://www.saint-venant-lab.fr/, sebastien.boyaval@saint-venant.enpc.fr.
[4] MATHICSE, Ecole Polytechnique Fédérale de Lausanne, Station 8, 1015 Lausanne, marco.picasso@epfl.ch, http//asn.epfl.ch/∼picasso

Key words: *Multiphase flow, Free surfaces, Volume-of-fluid, Time splitting algorithm, Immiscible phases, Landslides.*

We present a numerical method for the solution to the density-dependent incompressible Navier-Stokes equations modeling the flow of N immiscible incompressible liquid phases separated by interfaces and one additional gas phase (modeled by a vacuum) separated from the liquids by a free surface. The numerical modeling of such three-dimensional multi-phase flows involves thus a total of $N+1$ phases.

We use an Eulerian formulation to guarantee the mass conservation of all phases via a volume-of-fluid (VOF) approach involving N indicator functions (one per phase, identified by its density). The governing equations are thus the density-dependent Navier-Stokes equations for the velocity $\mathbf{u}$ and pressure p, together with N advection equations for the volume fractions φ_ℓ:

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u}\right) - \nabla\cdot(2\mu(\rho)\mathbf{D}(\mathbf{u})) + \nabla p = \rho\mathbf{f}$$

$$\nabla\cdot\mathbf{u} = 0$$

$$\frac{\partial \varphi_\ell}{\partial t} + \mathbf{u}\cdot\nabla\varphi_\ell = 0\,,\quad \ell = 1\ldots N$$

coupled with appropriate initial and boundary conditions, typically with a free force $-p\mathbf{n} + 2\mu(\rho)\mathbf{D}(\mathbf{u})\mathbf{n} = 0$ on the free surface between liquids and air.

The resulting system of partial differential equations is solved by means of a flexible operator splitting strategy that decouples advection and diffusion phenomena at each time step. Transport equations are first solved on a fine structured Cartesian grid with the forward characteristics method to predict the new location of each liquid phase. A particular emphasis is paid on the numerical approximation of the interfaces between liquid phases and of the free surface. To reduce the numerical diffusion around interfaces, a multiphase version of the SLIC algorithm is used on the Cartesian grid, while the artificial compression that jeopardizes the conservation of mass is reduced by means of a robust multiphase decompression algorithm.

A generalized Stokes problem, with a variable density and a density-dependent viscosity, is then solved with a piecewise linear finite element method on a coarser mesh of the liquid domain. More precisely, the variables are interpolated from the structured grid to the unstructured mesh, and the Stokes problem is solved only in the liquid domain.

Numerical experiments focus on hydraulic engineering benchmark cases, involving two or more liquid phases. In particular, we discuss the numerical simulation of impulse water waves, created by landslides falling into a lake. The examples treated involve two liquid phases and a gas (vacuum) phase. The straightforward extension to the modeling and simulation of solid-liquid interactions, by considering a rigid body as a *very viscous* liquid phase, is illustrated via numerical examples. Validation is achieved with computational results compared with experimental results.

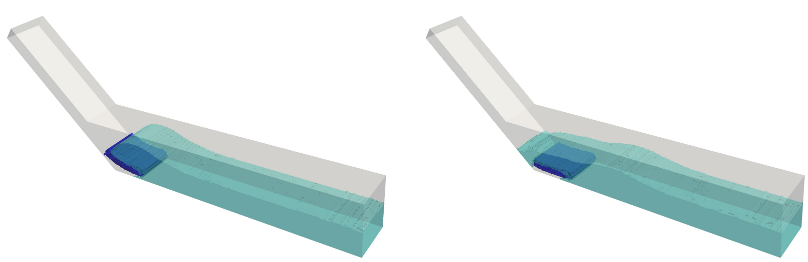

Figure 1: Wave generated by a landslide; snapshots of the simulation at times $t = 4$ and $t = 8$ [s].

REFERENCES

[1] A. Caboussat, M. Picasso, and J. Rappaz. Numerical simulation of free surface incompressible liquid flows surrounded by compressible gas. *J. Comput. Phys.*, 203(2):626–649, 2005.

[2] N. James, S. Boyaval, A. Caboussat, and M. Picasso. Numerical simulation of 3d free surface flows: the case of multiple incompressible liquid phases. *Int. J. Numer. Meth. Fluids*, submitted, 2013.

[3] V. Maronnier, M. Picasso, and J. Rappaz. Numerical simulation of three-dimensional free surface flows. *Int. J. Numer. Meth. Fluids*, 42(7):697–716, 2003.