11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
July 20–25, 2014, Barcelona, Spain

ADAPTIVE C^0 INTERIOR PENALTY METHOD FOR BIHARMONIC EIGENVALUE PROBLEMS

Susanne C. Brenner[1], **Joscha Gedicke**[*,2] **and Li-Yeng Sung**[3]

[1] Department of Mathematics and Center for Computation and Technology, Louisiana State University, Baton Rouge, LA 70803, USA, brenner@math.lsu.edu

[2] Department of Mathematics and Center for Computation and Technology, Louisiana State University, Baton Rouge, LA 70803, USA, jgedicke@math.lsu.edu

[3] Department of Mathematics and Center for Computation and Technology, Louisiana State University, Baton Rouge, LA 70803, USA, sung@math.lsu.edu

Key words: *eigenvalue, interior penalty method, biharmonic, a posteriori*

This talk presents a residual based *a posteriori* error estimator for biharmonic eigenvalue problems and the C^0 interior penalty method. Biharmonic eigenvalue problems occur in the analysis of vibrations and buckling of plates. The *a posteriori* error estimator is proven to be reliable and efficient for sufficiently large penalty parameter $\sigma \geq 1$ and sufficiently small global mesh size H. The theoretical results are verified in numerical experiments.

The C^0 interior penalty method [1, 3] avoids the use of complicated C^1 finite elements but uses standard Lagrange finite elements of total degree $k \geq 2$ for some triangulation $\mathcal{T}_\ell$. This method is nonconforming in the sense that $P_k(\mathcal{T}_\ell) \cap H^1_0(\Omega) := V_\ell \not\subset H^2_0(\Omega)$, and the associated nonconforming bilinear form is symmetric, continuous and coercive for sufficiently large penalty parameter $\sigma \geq 1$ [1, 3]. Similar to the *a posteriori* error estimator for the source problem [2], the *a posteriori* error estimator η_ℓ for the vibrating plate, with discrete eigenpair $(\lambda_\ell, u_\ell) \in \mathbb{R}_+ \times V_\ell$, set of edges $\mathcal{E}_\ell$, set of interior edges $\mathcal{E}^i_\ell$, jump $[\![\cdot]\!] = \cdot|_{T_+} - \cdot|_{T_-}$ across an edge $E = T_+ \cap T_-$, $T_\pm \in \mathcal{T}_\ell$, and polynomial degree $k = 2$, reads

$$\eta_\ell^2 := \sum_{T\in\mathcal{T}_\ell} h_T^4 \lambda_\ell \|u_\ell\|^2_{L_2(T)} + \sum_{E\in\mathcal{E}_\ell} \frac{\sigma^2}{h_E} \|[\![\partial u_\ell/\partial n]\!]\|^2_{L_2(E)} + \sum_{E\in\mathcal{E}^i_\ell} h_E \|[\![\partial^2 u_\ell/\partial n^2]\!]\|^2_{L_2(E)}.$$

The *a posteriori* error estimator η_ℓ is proven to be reliable and efficient up to higher order terms in some mesh-dependent norm. The reliability and efficiency of η_ℓ^2 for the eigenvalue error $|\lambda - \lambda_\ell|$ is verified in numerical experiments with varying penalty parameters, sizes of eigenvalues, and for convex and non-convex domains.

REFERENCES

[1] S.C. Brenner, *C^0 Interior Penalty Methods*, Frontiers in numerical analysis–Durham 2010, 79–147, Lect. Notes Comput. Sci. Eng., 85, Springer, Heidelberg, 2012.

[2] S.C. Brenner, T. Gudi, L.-Y. Sung, *An a posteriori error estimator for a quadratic C^0-interior penalty method for the biharmonic problem*, IMA J. Numer. Anal. **30** (2010), no. 3, 777–798.

[3] S.C. Brenner, L.-Y. Sung, *C^0 interior penalty methods for fourth order elliptic boundary value problems on polygonal domains*, J. Sci. Comput. **22/23** (2005), 83–118.