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IMPROVEMENT OF CHEAP APPROXIMATIONS BY A POST-PROCESSING/REDUCED BASIS RECTIFICATION METHOD

Yvon Maday[1], Olga Mula[2] and Benjamin Stamm[3]

[1] Laboratoire Jacques-Louis Lions (LJLL), UPMC Univ Paris 6, F-75005 Paris, France; and nstitut Universitaire de France; and Division of Applied Mathematics, Brown University, Providence, Rhode Island, USA; maday@ann.jussieu.fr

[2] LJLL, UPMC Univ Paris 6, F-75005 Paris, France; and CEA Saclay, DEN/DANS/DM2S/SERMA/LLPR, 91191 Gif-Sur-Yvette CEDEX, France; olga.mula@gmail.com

[3] LJLL, UPMC Univ Paris 6 and CNRS, F-75005 Paris, France; stamm@ann.jussieu.fr

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The aim of this communication is to shed light on a successful post-processing strategy first presented in [1] and then used in [2] in the framework of reduced basis simulation of PDE's. Some cheap and non-optimal reduced basis approximation is post-processed through some snapshots which allows to recover a very accurate approximation.

In a general framework, the main idea consists in the following: let $\mathcal{X}$ be a Banach space and let F be a compact subset of $\mathcal{X}$ of small Kolmogorov n-width (F can be, e.g., the set of solutions of a parameter dependent PDE as was the case in [1] and [2]). The goal is to accurately approximate any $f \in F$ by elements of a finite dimensional subspace $X_M \subset \mathcal{X}$ of small dimension M. Suppose that we have at our disposal two approximation operators:

- $\pi_M : \mathcal{X} \to X_M$ that provides a computationally expensive but accurate approximation of the elements of F, i.e. such that
$$\sup_{f\in F} \|f - \pi_M[f]\|_{\mathcal{X}}$$
is small enough for the application under consideration,
- $\mathcal{J}_M : \mathcal{X} \to X_M$ that provides a cheap but inaccurate approximation of the elements of F, i.e. such that $\sup_{f\in F} \|f - \mathcal{J}_M[f]\|_{\mathcal{X}}$ is not small enough for our standards.

The operators π_M and $\mathcal{J}_M$ can be Galerkin-projections as in [1] (finite element Galerkin projection) or [2] (reduced basis Galerkin projection) but we emphasize that we are placing ourselves in a much more general setting here. In this framework, we will discuss the hypothesis under which one can build from evaluations of $\mathcal{J}_M$ a rectification operator $\tilde{\pi}_M$ that has a comparable accuracy of π_M in the sense that

$$\sup_{f\in F}\|f-\tilde{\pi}_M[f]\|_{\mathcal{X}} \sim \sup_{f\in F}\|f-\pi_M[f]\|_{\mathcal{X}},$$

but that circumvents the computational cost of π_M.

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