

## A STABLE X-FEM IN COHESIVE TRANSITION FROM CLOSED TO OPEN CRACK.

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We focus on the problem of a solid 2D domain partitioned by a discontinuity, figure 1a. The discrete approach uses an X-FEM discretization to insert a mesh-independent embedded discontinuity [1]. Both sides of the discontinuity, the crack faces, are allowed to separate with a linear traction-separation law, a cohesive law. The linear cohesive law is governed by a constitutive interface stiffness. Particular cases are the open crack, with zero interface stiffness, and the closed crack, that can be regarded as a constraint with “infinite” interface stiffness.

The stability of the X-FEM discretization is trivial by construction for the open crack [2], but remains a challenge if we constrain the crack closed, as in the bi-material problem [3]. Here, we develop an embedded formulation for general cohesive interactions between crack faces. The formulation is based on the mixed enriched displacement-stress formulation of Zilian and Fries [1], the stable X-FEM. An additional field, the crack opening, is introduced in a three-field formulation. The stability is studied for any linear crack stiffness adapting the arguments of Baiges et al [4], and can also be generalized to any discretization fulfilling certain relatively weak conditions.

We perform a benchmark with similar approaches, for closed crack (Lagrange multiplier [3]), for linear cohesive crack (penalty approach), figure 1b. Also, a nonlinear cohesive softening is illustrated [5] [6]. The proposed method has additional advantages with respect to state-of-the-art closed and open crack models. Due to the analogies to stable X-FEM and Nitsche’s methods [7], we observe that the method simplifies the implementation and is attractive in dynamic explicit codes.

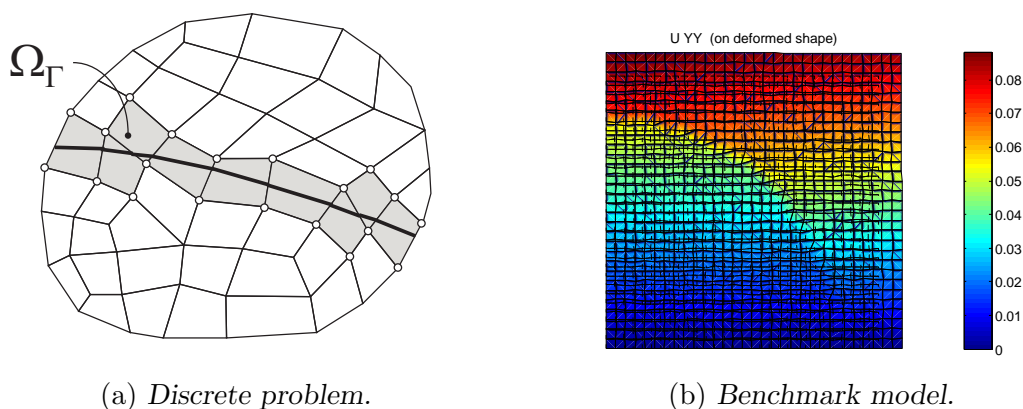


Figure 1: (a) Discrete domain with an embedded mesh-independent discontinuity. The enriched X-FEM dof's are represented as hollow circles, and the mixed variables are defined elementwise on the shaded elements. (b) Benchmark problem with 1/4 symmetry: pressure load with a circular interface in a distorted quadrilateral mesh. The vertical displacement  $U_{YY}$  is discontinuous, with a displacement jump controlled by a cohesive stiffness.

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