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SIMULATION OF ONE-DIMENSIONAL FLOW IN VISCOELASTIC TUBE

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The blood flow in the human arterial tree may be considered as one-dimensional flow in a network of viscoelastic blood vessels. One-dimensional models present a good compromise between Windkessel and three-dimensional models [1]. Here we apply the method of characteristics to simulate one-dimensional blood flow in a pipe with elastic and viscoelastic wall and mathematical model reads:

$$\frac{\partial A}{\partial t}+\frac{\partial Q}{\partial x}=0 \qquad (1)$$

$$\frac{\partial Q}{\partial t}+\frac{A}{\rho}\frac{\partial p}{\partial x}+\frac{\partial (Qv)}{\partial x}=-fQ \qquad (2)$$

$$p-p_0=\frac{1}{C}(A-A_0)+\eta\frac{\partial A}{\partial t} \qquad (3)$$

x, t – the space and time coordinate, respectively,
A – the cross-sectional area ($A=D^2\pi/4$),
Q – volume flow rate, and $v=Q/A$,
C – the areal compliance of the wall ($C=\mathrm{d}A/\mathrm{d}p$),
η – the viscous resistance of the wall,
ρ – the fluid density,
f – the friction coefficient.

Method of characteristics

The artery is discretized into a number of elements of length Δx. Fig. 1 shows two typical elements (denoted by i and j) bounded by nodes (W, P, E). The pressure is defined at the nodes, A is defined in the middle of each element and Q is defined at each end of each element. Thus, four unknowns are stored at each node, as depicted in Fig. 1 for node P.

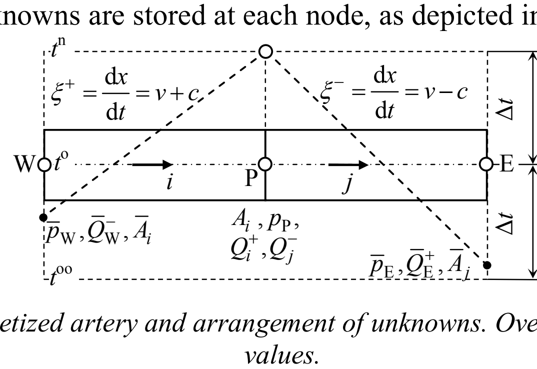

Figure 1 A part of discretized artery and arrangement of unknowns. Over bar denotes interpolated values.

Equations (1) to (3) may be transformed into ordinary differential equations that are valid along characteristics defined by $\xi^+=\mathrm{d}x/\mathrm{d}t=v+c$ and $\xi^-=\mathrm{d}x/\mathrm{d}t=v-c$, in the form:

$$\frac{A}{\rho}\mathrm{d}p + \xi^{\pm}\mathrm{d}Q - v^2\mathrm{d}A = -fQ\Delta x - \eta\frac{A}{\rho}\frac{\partial^2 Q}{\partial x \partial t}\mathrm{d}t \qquad (4)$$

where $c=\sqrt{A/(\rho C)}$ is the wave speed. Along the third characteristic (applied for each element), defined by ξ^0=dx/dt=0, the following relationship holds:

$$\mathrm{d}p - \frac{1}{C}\mathrm{d}A = -\eta\frac{\partial^2 Q}{\partial x \partial t}\mathrm{d}t \qquad (5).$$

The fourth equation related to each node is the continuity equation $Q_i^+=Q_j^-$. Equations (4) to (5) are discretized by replacing differentials with finite differences, e.g. for pressure along the ξ^+ characteristic: $\mathrm{d}p = p_\mathrm{P}^\mathrm{n} - \overline{p}_\mathrm{E}$. To obtain a linear system of equations, all coefficients in these equations are discretized using known variable values from previous times (t^o, t^oo, …).
The described method was applied to a tube of uniform cross section area A_0=20 cm^2 (at pressure p_0=0), divided into 25 equal elements of length Δx=8 cm. The input pressure is defined by the equation $\{p_\mathrm{in}\}_\mathrm{mmHg} = 100\sin(2\pi t/T)$, where T=1 s is the period. Boundary condition at the pipe outlet is defined by terminal resistance R. We consider two cases: a pure elastic (η=0) and a viscoelastic tube wall with η defined by the time constant τ=C·η=0.5 s. In both cases the integration step was T/100 and the total integration time was 30T. We choose terminal resistance R=1 mmHg·s/ml, c=6 m/s and ρ is selected to match R with characteristic impedance $Z_0=\sqrt{\rho/(A_0C)}=\rho c/A_0$. In the first case there are no wave reflections in the tube and the pressure and flow are in phase at each point of the tube. Fig. 2 shows the prescribed input pressure, calculated input flow and area (at the middle of the first element i.e. 4 cm from the pressure node). It is clear that the pressure and flow are in phase, and that they have the same amplitudes according to the analytical solution. The area is also in phase with the pressure (a small difference occurs because the pressure and area are not taken form the same point). Fig. 3 shows the analogue results for the viscoelastic case, where the phase lags of flow and area are visible, as well as their amplitude reduction.

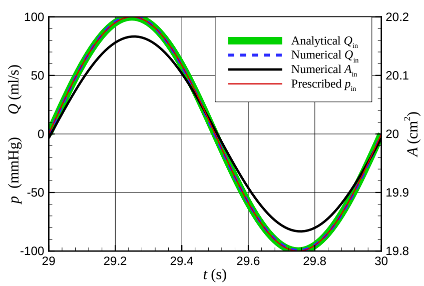

Figure 2 Input flow and pressure and first element area variations during the last integration period (elastic case)

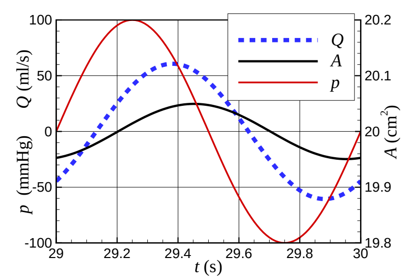

Figure 3 Input flow, pressure and area variations during the last integration period (the viscoelastic case)

In the case of viscoelastic tube, the term representing the wall viscosity appears in the source term (right hand side of characteristic equation), but the developed method of characteristics is still physically clear and efficient as in the case of an elastic tube.

REFERENCES

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