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A Unified Hybridized Discontinuous Galerkin Framework and Its Application to PDEs

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By revisiting the basic Godunov approach for linear system of hyperbolic Partial Differential Equations (PDEs) we show that it is hybridizable. As such, it is a natural recipe for us to constructively and systematically establish a unified HDG framework for a large class of PDEs including those of Friedrichs' type. The unification is fourfold. First, it provides a single constructive procedure to devise HDG schemes for elliptic, parabolic and hyperbolic PDEs. Second, it reveals the nature of the trace unknowns as the Riemann solutions. Third, it provides a parameter-free HDG framework, and hence eliminating the ``usual complaint" that HDG is a parameter-dependent method. Fourth, it allows us to construct the existing HDG methods in a natural manner. In particular, using the unified framework we can rediscover most of the existing HDG methods and furthermore discover new ones.

We apply the proposed unified framework to three different PDEs: the convection-diffusion-reaction equation, the Maxwell equation in both frequency and time domains, and the Stokes equation. The purpose is to present a step-by-step construction of various HDG methods, including the most economic ones with least trace unknowns, by exploiting the particular structure of the underlying PDEs. The well-posedness of the resulting HDG schemes, i.e. the existence and uniqueness of the HDG solutions, are proved. The well-posedness results are also extended and proved for abstract Friedrichs' systems. We also discuss variants of the proposed unified framework and extend them to the popular Lax-Friedrichs flux and to nonlinear PDEs. Numerical results for transport equation, convection-diffusion equation, compressible Euler equation, and shallow water equation are presented to support the unification of the HDG methods.