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A DISCONTINUOUS GALERKIN LOCAL ORTHOGONAL DECOMPOSITION METHOD FOR ELLIPTIC MULTISCALE PROBLEMS

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We study the numerical approximation of elliptic problems with possible strong convection and with heterogeneous and highly varying data without any assumptions on scale separation or periodicity. Problems of this type arise in many branches of scientific computing, for example in porous media flow and in composite materials. There are two reasons why standard continuous finite element methods perform poorly for this kind of problems. That is, both the data describing the problem as well as boundary and internal layers in the solution needs to be resolved. We propose a discontinuous Galerkin local orthogonal decomposition (DG-LOD) method [2, 3], based on a corrected basis calculated on localized patches. The corrected basis functions takes the fine scale variations in the data into account. Let u_H^{ms} be the solution obtained by the DG-LOD method on a coarse mesh of size H which does not resolve the data. Then the following result holds

$$|||u - u_H^{\mathrm{ms}}||| \leq |||u - u_h||| + CH,$$

under moderate assumptions on the magnitude of the convection and that the size of the patches where the corrected basis is computed on are chosen as $\mathcal{O}(H \log(H^{-1}))$. The constant C is independent of the variation in the data and of the mesh size, and u_h is the (one scale) discontinuous Galerkin solution computed on the same scale as the corrected basis functions. We will also consider how to further reduce the computational cost using adaptivity [1]. The result holds independent of the regularity of the solution u.

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