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SUPPRESSION OF VIBRATION IN BOUNDED STRUCTURES SUBJECTED TO ACTION OF A DISTRIBUTED LOAD BY CONTINUOUS SPATIAL MODULATIONS OF THEIR PARAMETERS

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The paper is concerned with a problem of vibration suppression in any preassigned region of a bounded structure subjected to action of an external load which is distributed over its entire area. Continuous spatial modulations of structure parameters, particularly of its material, are proposed to be used as a means for vibration suppression. The considered problem is relevant, in particular, for the task of sound and vibration isolation which gained much attention in the recent years (see, e.g. [1-4]). Usually, behaviour of unbounded structures in presence of a point load or an excitation source is considered within this task [1,2]. So, the problem reduces to identification of frequency stop and pass bands and their subsequent tailoring by spatial/temporal modulations of system parameters. In the present paper the problem of vibration suppression is considered in a different formulation: behaviour of a bounded structure under action of a distributed load is studied. Similar cases were analyzed, in particular, in the monograph [3]. We note that the problem under study is close to those solved by the method of topology optimization [4], which is a popular method for obtaining the optimal layout of one or several material constituents in structures and materials.

As a simple illustrative example, consider oscillations of a string under action of an external distributed time-periodic load. The suppression of vibration in predefined areas of the string is carried by continuous spatial modulation of its cross-section. So, the following equation is studied:

$$\rho S \frac{\partial^2 u}{\partial t^2} - \frac{\partial}{\partial x}\left(T \frac{\partial u}{\partial x} \right) = f(x,t) , \qquad (1)$$

here ρ is density of the string material, T is tension force, $u(x,t)$ is lateral deflection of the string, $S(x)$ is the variable cross-sectional area, $f(x,t) = f_0(x)\sin\omega t$ is density of the external load. The boundary conditions are set to be homogeneous: $u|_{x=0} = u|_{x=l} = 0$. To solve the considered problem a novel approach based on the method of direct separation of motions [5] is employed. First, the case of harmonic in the spatial coordinate x external load $f_0(x) = F_0 \cos kx$ was studied. As the result, it is shown that the optimal characteristics of vibration suppression will be achieved when cross-section is modulated in the following way:

$$S(x) = S_0(1 + \chi\cos(2kx + \theta)) , \qquad (2)$$

here $\theta=\pi$ if $\delta=\omega^2\rho S_0/(Tk^2)<1$, and $\theta=0$ if $\delta>1$. For example, it is possible to completely suppress vibration near one of the string ends. Also a considerable reduction of vibration near any preassigned point of the string can be achieved. The corresponding dependencies of the string deflection on the spatial coordinate x are presented in Figure 1 (a)-(c) for various χ and the following values of other parameters $\rho S_0/T=1(\mathrm{s/sm})^2$, $l=5\,\mathrm{sm}$, $k=10\;1/\mathrm{sm}$, $F_0=-10\,\mathrm{kg/s^2}$, $\omega=9.51/\mathrm{s}$. We note that it is possible to suppress vibration considerably in the whole string when χ is comparatively large (see Figure 1 (d)).

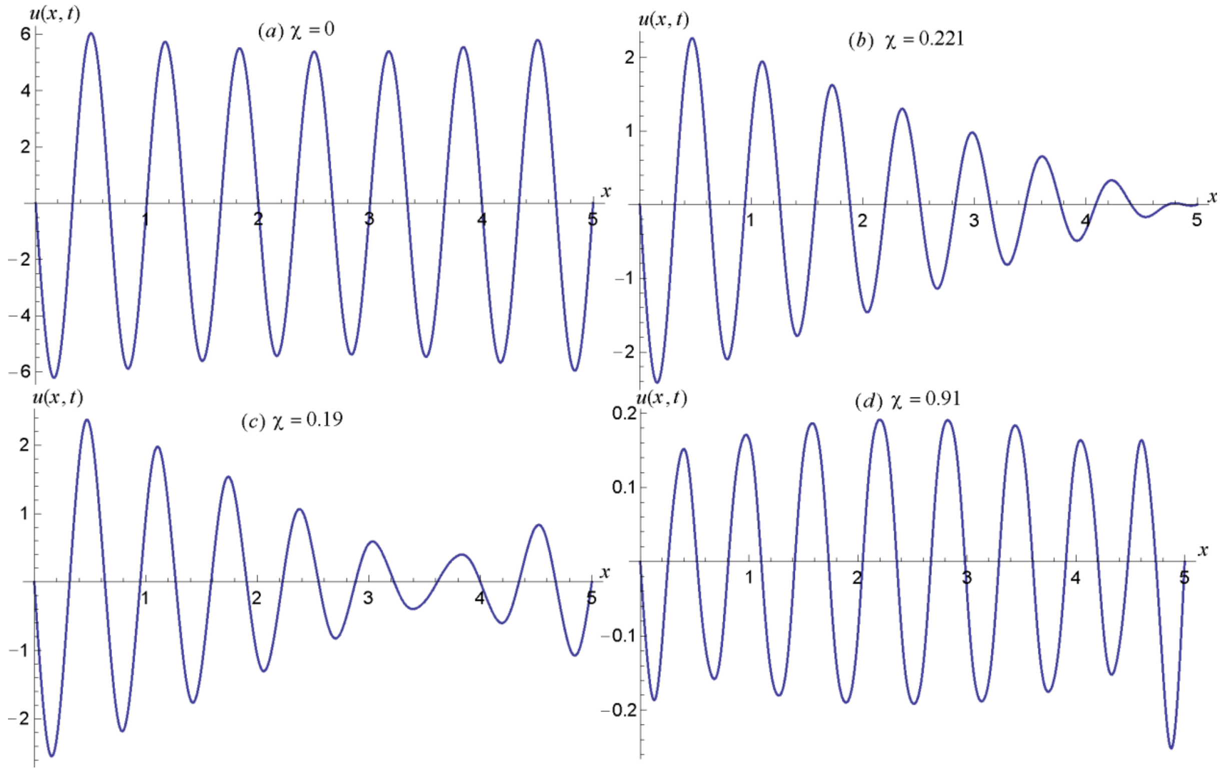

Figure 1. Dependencies of the string deflection on the spatial coordinate x.

The case of arbitrarily distributed external load was also studied. Expanding function $f_0(x)$ in the Fourier series, an optimal modulation of the string cross-sectional area is determined.

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