11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
July 20–25, 2014, Barcelona, Spain

GENERATION AND VALIDATION OF CURVED MESHES FOR UNSTRUCTURED HIGH-ORDER METHODS

A. Gargallo-Peiró[1], X. Roca[2], J. Peraire[3] and J. Sarrate[4]

[1,4] Laboratori de Càlcul Numèric (LaCàN), Universitat Politècnica de Catalunya, Barcelona 08034, Spain; abel.gargallo@upc.edu, jose.sarrate@upc.edu
[2,3] Department of Aeronautics and Astronautics, Massachusetts Institute of Technology, Cambridge, MA 02139, USA; xeviroca@mit.edu, peraire@mit.edu

Key words: *CAD, Distortion measure, Mesh optimization, Curved mesh, High-order*

In the last two decades, unstructured high-order methods have attracted a remarkable attention from the community of computational methods. For problems with smooth solutions, high-order methods have the potential to deliver higher accuracy with a lower computational cost than low-order methods [1, 2]. However, their application has been hampered by technical issues such as the difficulty of generating valid 3D curved meshes.

The main goal of this work is to propose a robust procedure to validate and generate curved and high-order meshes. The resulting meshes are composed by valid and high-quality elements with the boundary nodes on the surfaces of a CAD model. To this end, we use an *a posteriori* approach [3]. First, a linear mesh that prescribes the geometrical features desired for the high-order mesh is generated, see Figure 1(b). Second, the linear mesh is converted to a high-order mesh by increasing the interpolation degree. Third, a valid high-order surface mesh is generated. Specifically, the distortion of the surface mesh is minimized by moving the nodes on the CAD surfaces [4, 5]. Following, we curve the faces of the boundary elements to match the surface mesh, see Figure 1(c). In this step, non-valid (folded) and low-quality (distorted) elements can appear close to the curved boundaries. Finally, see Figure 1(d), the tetrahedral mesh is untangled and smoothed to obtain a mesh composed by valid and high-quality elements [6].

To implement the detailed procedure, two main ingredients are required. First, we propose a mesh quality measure to validate nodal high-order triangle and tetrahedral elements of any interpolation degree. The proposed measure is obtained as the extension of any Jacobian based distortion measure for linear elements. Second, we propose a robust smoothing and untangle method that converts an initial linear mesh (ideal mesh) to the desired curved and high-order mesh. The proposed method is based on a regularized and non-linear minimization of the mesh distortion. This minimization is formulated both for surface and 3D high-order meshes. The main advantage is that the minmization repairs

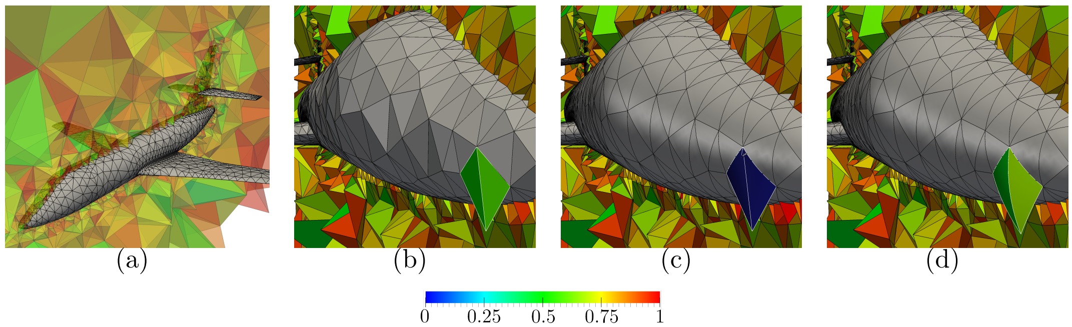

Figure 1: Tetrahedral meshes of interpolation degree 4 colored according to the shape quality measure on a Falcon aircraft: (a) overview of the final mesh, (b) initial straight-sided mesh, (c) initial curved mesh, and (d) final smoothed mesh.

non-valid configurations (untangles), since it uses an element-wise regularization of the mesh distortion.

Finally, we present several examples to illustrate the advantages and the application of the proposed technique. The examples show that the method can: untangle meshes composed by a large number of inverted elements; deal with convex features and large deformations; maintain the size prescribed on the linear mesh and preserve straight edges where possible; generate high-order boundary layer meshes; and be applied to generate valid and high-quality high-order tetrahedral meshes from CAD models.

REFERENCES

[1] P.E. Vos, S. Sherwin and R. Kirby. *From h to p efficiently: implementing finite and spectral/hp element methods to achieve optimal performance for low- and high-order discretisations*. J. Comput. Phys., Vol. **229**, Num. **13**, 5161–5181, 2010.

[2] A. Huerta, A. Angeloski, X. Roca and J. Peraire. *Efficiency of high-order elements for continuous and discontinuous Galerkin methods*. Int. J. Numer. Methods Eng., DOI = 10.1002/nme.4547.

[3] S. Dey, M.S. Shephard, and J.E. Flaherty. *Geometry representation issues associated with p-version finite element computations*. Comput. Meth. Appl. M., Vol. **150**, Num. **1–4**, 39–55, 1997.

[4] A. Gargallo-Peiró, X. Roca, and J. Sarrate. *A surface mesh smoothing and untangling method independent of the CAD parameterization*. Comp. Mech., DOI=10.1007/s00466-013-0920-1.

[5] A. Gargallo-Peiró, X. Roca, J. Peraire and J. Sarrate. *Defining Quality Measures for Mesh Optimization on Parameterized CAD Surfaces*. Proc. 21st Int. Meshing Roundtable, 85-102, 2013.

[6] A. Gargallo-Peiró, X. Roca, J. Peraire and J. Sarrate. *Defining quality measures for validation and generation of high-order tetrahedral meshes*. Proc. 22nd Int. Meshing Roundtable, 109-126, 2014.