

## FINITE BEAM ELEMENT WITH PIEZOELECTRIC LAYERS AND FUNCTIONALLY GRADED MATERIAL OF CORE

Vladimír Kutíš<sup>1</sup>, Justín Murín<sup>1</sup>, Juraj Paulech<sup>1</sup> and Juraj Hrabovský<sup>1</sup>

<sup>1</sup> Department of Applied Mechanics and Mechatronics, Institute of Automotive Mechatronics, FEI STU Bratislava, *vladimir.kutis@stuba.sk*

**Key Words:** *FGM, beam finite element, piezoelectric material.*

New materials have been introduced in the area of mechanisms and applications. Contemporary mechanical and mechatronic systems are focusing on minimizing size, active control and low energy consumption. All this attributes can be incorporated into term Micro Electro Mechanical Systems (MEMS) [1]. To improve performance of MEMS system, new materials and technologies are developed - one of them, which found broad application usage is Functionally graded material (FGM). MEMS application usually contains multi layers structure and in some application class MEMS systems contain piezoelectric layers. Piezoelectric structures offer facilities to make motions. Piezoelectric layers can be also used to damp vibrations as an active damping or as an active sensor. For better understanding these multiphysical problems new mathematical models are developed.

To obtain basic FEM equations for piezoelectric material, principle of virtual work is used and basic FEM equations of piezoelectric element has form

$$\begin{bmatrix} \mathbf{K}_{uu}^e & \mathbf{K}_{u\varphi}^e \\ \mathbf{K}_{\varphi u}^e & \mathbf{K}_{\varphi\varphi}^e \end{bmatrix} \begin{bmatrix} \mathbf{u}^e \\ \boldsymbol{\varphi}^e \end{bmatrix} = \begin{bmatrix} \mathbf{F}^e \\ \mathbf{Q}^e \end{bmatrix} \quad (1)$$

where  $\mathbf{K}_{uu}^e$  is structural submatrix,  $\mathbf{K}_{\varphi\varphi}^e$  is electric submatrix and  $\mathbf{K}_{u\varphi}^e$  and  $\mathbf{K}_{\varphi u}^e$  are coupling submatrix of element.  $\mathbf{F}^e$  and  $\mathbf{Q}^e$  are element nodal forces and charges, respectively.  $\mathbf{u}^e$  and  $\boldsymbol{\varphi}^e$  are element nodal displacements and potentials, respectively.

Fig. 1 shows piezoelectric beam element where core is made of functionally graded materials. Top and bottom layers are made of piezoelectric materials. Beam finite element has 6 structural degree of freedom ( $u_i, v_i, \phi_i, u_j, v_j, \phi_j$ ) and 4 electrical degree of freedom ( $\varphi_1, \varphi_2, \varphi_3, \varphi_4$ ). Structural loading is represented by 4 forces and 2 moments ( $F_{xi}, F_{yi}, M_i, F_{xj}, F_{yj}, M_j$ ) and electrical loading by 4 charges ( $Q_1, Q_2, Q_3, Q_4$ ).

The paper deals with homogenization of FGM material properties - where direct integration method [2] is used, and homogenization of core and piezoelectric layers - where multilayer method [3] is used. In the paper, there is presented the derivation of individual submatrices of local stiffness matrix, where concept of transfer constants [4] is used. Functionality of new FGM finite beam with piezoelectric layers is presented by numerical experiments.

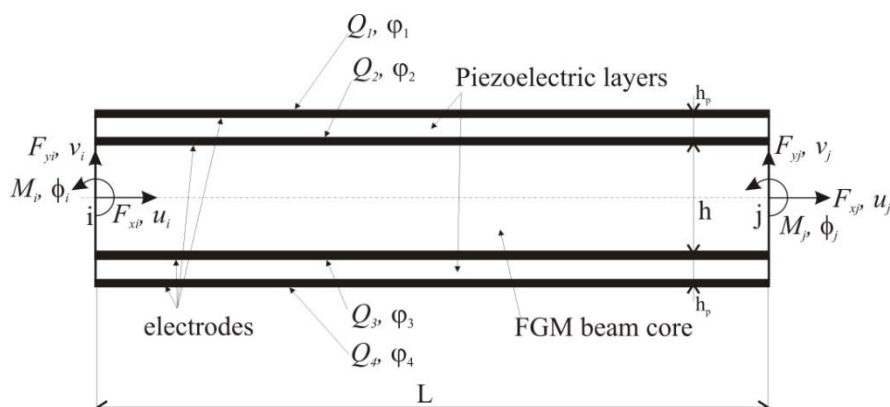


Fig.1: Piezoelectric beam element with FGM core

## REFERENCES

- [1] N. Maluf: An Introduction Microelectromechanical Systems Engineering, Artech House, 2000.
- [2] V. Kutiš, J. Murín, R. Belák, J. Paulech: Beam element with spatial variation of material properties for multiphysics analysis of functionally graded materials, *Computers & Structures*, 89, 2011.
- [3] Murin J, Kutiš V, Masný M, Duriš R. Composite (FGMs) beam finite elements. In: Kompiš V, editor. Composites with micro and nano-structure. Springer Science + Business Media B.V.; 2008.
- [4] Rubin H. Bautechnik. Berlin: Ernst and Sohn; 1999.

## ACKNOWLEDGEMENT

This work was supported in part by the following projects: Slovak Research and Development Agency under the contracts APVV-0450-10, APVV-0246-12, Grant Agency KEGA - grant No. 015STU-4/2012 and VEGA - grant No. 1/0534/12 and 1/0228/14.