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A DIRECETIONAL FAST MULTIPLOLE METHOD FOR THE BOUNDARY ELEMENT METHOD AND ITS APPLICATION TO ELASTODYNAMICS

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The Boundary Element Method (BEM) is widely used in many different engeneering disciplins. Its applicability however suffers from the arising dense system matrices requiring a storage amount of $O(N^2)$ (N being the number of degrees of freedom (DOF)). Without the use of fast methods it is thus limited to medium sized problems. Using BEM for time dependent problems poses another problem since the computation of the time convolution requires the storage of dense system matrices for every timestep M. To overcome the latter problem we use a modified version of the convolution quatdrature method (CQM) where only one system matrix is computed at the time. The Adaptive Cross Approximation (ACA) and a directional fast multipole method (DFMM) is then used to further reduce computational costs and storage requirements.

As mentioned above, the convolution in time is performed with the reformulated version of the CQM introduced by Banjai et. al [2]. This allows us to split the convolution into decoupled problems in Laplace domain with the complex Laplace parameter s_l. The elastodynamic fundamental solution for the displacements in Laplace domain is

$$\hat{U}_{ij}(r,s_l) = A^P_{ij}(r,s_l)e^{\mathrm{i}k_P r} + A^S_{ij}(r,s_l)e^{\mathrm{i}k_S r},$$
$$A^P_{ij}(r,s) = \frac{e^{-d_P r}}{4\pi\rho s_l^2}\left(\frac{3r_{,i}r_{,j}-\delta_{ij}}{r^3}\left(\frac{s_l}{c_P}r+1\right)+\left(\frac{s_l}{c_P}\right)^2\frac{r_{,i}r_{,j}}{r}\right),$$
$$A^S_{ij}(r,s) = \frac{e^{-d_S r}}{4\pi\rho s_l^2}\left(\frac{3r_{,i}r_{,j}-\delta_{ij}}{r^3}\left(\frac{s_l}{c_S}r+1\right)+\left(\frac{s_l}{c_S}\right)^2\frac{r_{,i}r_{,j}+\delta_{ij}}{r}\right)$$

with $d_{P/S} = \Re(\frac{s_l}{c_{P/S}})$ and $k_{P/S} = \Im(\frac{s_l}{c_{P/S}})$. Obviously, the complex Laplace parameter s_l causes both an oscillation and a damping of the fundamental solution. The used Laplace parameters in CQM lie on an ellipse in the complex plane where we can broadly identify two regions. A region with a large real part, i.e. high damping and low oscillation of $\hat{U}_{ij}(r, s_l)$, where the ACA can be used to compress the arising system matrices. Second, a region with small real and large imaginary values of s_l, i.e. strong oscillations arise where a DFMM is used to speed up the computation.

A directional version of the black-box fast multipole method, presented in [1], was introduced by Messner et al. [3] for the Helmholtz kernel. Here, this idea is extend to elastodynamics. This can either be done by directly approximating the tensorial fundamental solution or by rewriting it using derivatives of the Helmholtz kernel. A comparison of both methods in terms of storage efficiency and computatioal costs is presented.

As a numerical example we us a rod of dimensions $3\,\mathrm{m} \times 1\,\mathrm{m} \times 1\,\mathrm{m}$ with a heavyside traction loading on one end. The other end is clamped. We use the collocation method for the mixed boundary value problem. The numerical results are compared to the $1D$ solution.

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