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THE NURBS-ENHANCED FINITE ELEMENT METHOD (NEFEM) FOR FREE-SURFACE FLOW SIMULATIONS

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The handling of transient free-surface flows is crucial for relevant physical phenomena as well as engineering applications. A particular example in the field of plastics processing is the manufacturing process of profile extrusion. This process aims to form continuous profiles with a fixed cross-section out of an originally circular melt strand. The melt flows through an extrusion die with an outflow shape derived from the profile shape. After leaving the outflow, the melt flows freely (free-surface flow) and swells due to viscoelastic effects. Simulating this phenomenon with the finite element method is a daunting challenge, because the accuracy depends mainly on geometric quantities like normal vectors and curvatures. They have to be derived from the available discretized finite element mesh, which is often only a piecewise linear interpolation of the physical geometry. Hence, regardless of the complexity of the flow solution itself, a high degree of mesh refinement is needed to properly approximate the domain geometry. This is ineffective and increases the computational cost even for simple problems. The remedy is to use better geometry representation to handle geometry easily and at the same time more accurately.

One computational concept, which guarantees geometrical accuracy, is Isogeometric Analysis (IGA) [1], where the same basis functions that are used for the generation of the geometric model are applied for the analysis. In our case, this means that the same complex and expensive to compute shape functions used for the geometry representation are unnecessarily also used for the flow solution. Considering that the crucial geometry information for free-surface flows is limited to the moving boundary, it is possible to combine the advantages of IGA and standard FEM by introducing an exact description of the free-surface boundary to the standard finite element mesh. Elgeti *et al.* reported first steps in this direction in 2011 by incorporating a NURBS parameterization of the free surface, which is independent of the finite element discretization of the domain and is used to compute geometric quantities [2]. The concept can be further improved by including the NURBS parameterization in the element formulation, which leads to a grid with two element types: standard finite elements using isoparametric polynomial interpolation both in the interior of the domain and on the non-moving boundaries and mixed elements on the free surface. The mixed elements break with the isoparametric concept and use polynomial approximation for the solution field and

NURBS shape functions for the geometry description. Therefore, special treatment is required for these elements, but since their number is comparatively small, there is no significant increase in the computational effort. At the same time, geometric quantities on the moving boundary are computed precisely regardless of the actual finite element discretization. This idea is an extension of the NURBS-enhanced finite element method proposed by Sevilla *et al.* in [3]. For the free-surface flow simulation, the method is implemented within the framework of the in-house flow solver XNS, which uses the Galerkin/Least-Squares stabilization technique in a space-time finite element setting. A comparison between the standard FEM approach for computing geometric quantities and the NEFEM methodology for a 2D simulation of die swell is outlined in Fig. 1.

Topics discussed will be the theoretical background of NEFEM with its differences and difficulties compared to standard FEM. Special attention with respect to conservation of mass will be paid to the method for moving the free surface for transient analysis. Benchmark tests and recent simulations of free-surface flows will be shown. They will assess the performance of NEFEM and give insight into the influence of the NURBS geometry representation.

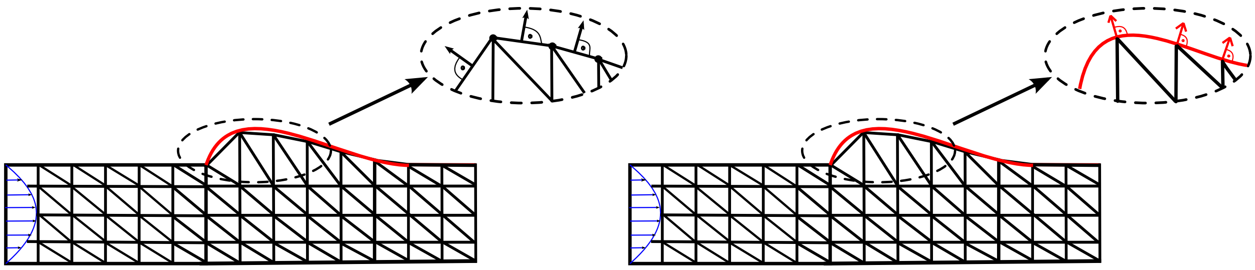

Figure 1. Difference between computing normal vectors on the free surface, using the linear actual discretization in the standard FEM context on the left and using the NURBS boundary representation on the right.

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