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GOAL-ORIENTED A POSTERIORI ERROR ESTIMATION IN STABILIZED DISCRETIZATIONS OF CONVECTION-DIFFUSION-REACTION MODELS

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Numerical simulations of convection-diffusion-reaction phenomena are desirable in several fields of natural sciences and in a large number of branches of technology. As a protoype model we study the equation

$$\partial_t u + \boldsymbol{b} \cdot \nabla u - \nabla \cdot (\varepsilon \nabla u) + r(u) = f\,, \tag{1}$$

equipped with appropriate initial and boundary conditions. In applications of practical interest, the size of the diffusion in (1) is often smaller by several orders of magnitude compared to the size of the flow field. Then, Eq. (1) admits solutions with sharp moving fronts, interior or boundary layers and complicated structures where important physical and chemical phenomena take place. The accurate and reliable numerical approximation of solutions to Eq. (1) continues to be a challenging task.

In this contribution we consider approximating solutions of Eq. (1) by lower and higher order conforming finite element methods with streamline upwind Petrov-Galerkin (SUPG) stabilization. In addition, discontinuity- or shock-capturing stabilization as an additional consistent modification of the numerical scheme and stabilization in crosswind direction is applied; cf. [2]. These types of schemes have demonstrated in comparative studies of stabilized discretizations their potential to compute robust approximations in applications where sharpness and position of layers are of importance. Nevertheless, these techniques are generally not perfect since the computations are often perturbed by spurious oscillations in a vicinity of layers.

To improve further the approximation quality and efficiency of the schemes, we combine the stabilized discretizations with an a posteriori error control mechanism and an adaptive mesh generation algorithm based on the dual-weighted-residual approach; cf. [1]. The dual

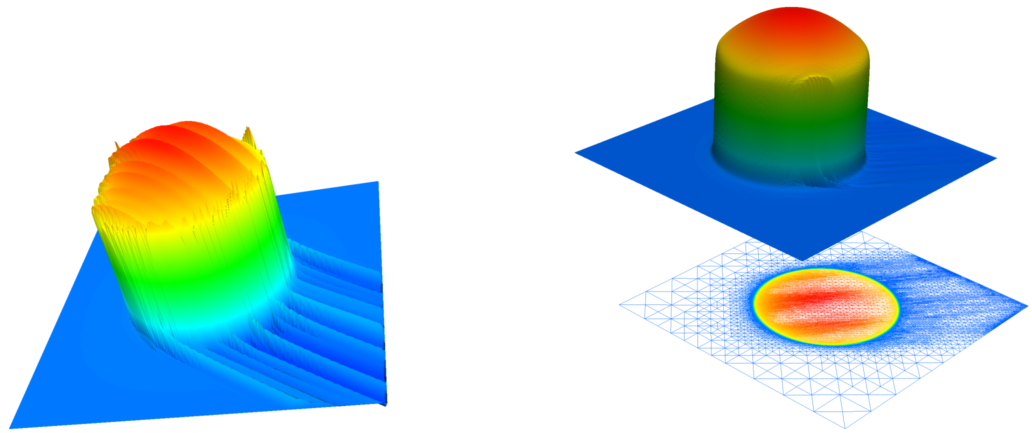

Figure 1: Stabilized $\mathcal{P}_1$ finite element approximation of the stationary hump problem [2, 3] on a uniform (*left*) and on an adaptively refined mesh (*right*) with $\varepsilon = 10^{-6}$.

weighted residual method is capable to assess the discretization error with respect to a given quantity of physical interest and not only with respect to the energy norm or the norm associated with the underlying discretization. Even though it seems being natural to apply adaptive finite element techniques to transport problems with sharp fronts and layers, combined stabilized and adaptive finite element methods have been studied only in a few contributions to the literature so far. One reason for this might be the difficulty to derive robust a posteriori error bounds in which the occuring constants do not depend on the small diffusion parameter ε; cf. [3].

We study different approaches for combining SUPG and shock-capturing stabilization with dual weighted residual error estimation. On the one hand, the formal adjoint of the given problem is used for the dual problem and stabilization of the primal and dual problem is applied afterwards. On the other hand, the transpose of the bilinear form for the stabilized method is chosen as the adjoint problem of the a posteriori error analysis. Various output functionals of the solution are proposed and studied. By numerical experiments we analyze and illustrate the efficiency and numerical performance properties of the algorithms; cf. Fig. 1.

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