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3D MEMBRANE THEORY (ABSTRACT)

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Membrane structures are characterized by tangential in-plane structural tensile stress state [1]. Tensile force equilibrium is described in membranes' deformed curved configuration. The presented numeric model shows analytic description of force equilibrium by a virtual work approach that applies 2nd Piola-Kirchhoff stress and Green-Langrange strain referred to the undeformed stress free configuration as well as Cauchy stress and Euler-Almansi strain referred to the deformed configuration [2]. The weak form of the problem is expressed via spatial integration over the domain of the considered structure [3]. By parametric ξ–η description of the undeformed initial geometric membrane surface $[{}^{0}\mathrm{x}(\xi,\eta);\ {}^{0}\mathrm{y}(\xi,\eta);\ {}^{0}\mathrm{z}(\xi,\eta)]$ and the unknown translational displacements $[{}^{t+\Delta t}\mathrm{u}_x(\xi,\eta);\ {}^{t+\Delta t}\mathrm{u}_y(\xi,\eta);\ {}^{t+\Delta t}\mathrm{u}_z(\xi,\eta)]$ that give the deformed state of the membrane at $t+\Delta t$ the geometric description is unique with membrane surface tangential vectors ${}^{0}\mathrm{g}_\xi(\xi,\eta)=\partial[{}^{0}\mathrm{x}(\xi,\eta)+0;\ {}^{0}\mathrm{y}(\xi,\eta)+0;\ {}^{0}\mathrm{z}(\xi,\eta)+0]/\partial\xi$, ${}^{0}\mathrm{g}_\eta(\xi,\eta)=\partial[{}^{0}\mathrm{x}(\xi,\eta)+0;\ {}^{0}\mathrm{y}(\xi,\eta)+0;\ {}^{0}\mathrm{z}(\xi,\eta)+0]/\partial\eta$, ${}^{0}\mathrm{g}_{normal}(\xi,\eta)={}^{0}\mathrm{g}_\xi(\xi,\eta)\times{}^{0}\mathrm{g}_\eta(\xi,\eta)$ with reference to the undeformed state of the membrane at 0 and
${}^{t+\Delta t}\mathrm{g}_\xi(\xi,\eta)=\partial[{}^{0}\mathrm{x}(\xi,\eta)+{}^{t+\Delta t}\mathrm{u}_x(\xi,\eta);\ {}^{0}\mathrm{y}(\xi,\eta)+{}^{t+\Delta t}\mathrm{u}_y(\xi,\eta);\ {}^{0}\mathrm{z}(\xi,\eta)+{}^{t+\Delta t}\mathrm{u}_z(\xi,\eta)]/\partial\xi$,
${}^{t+\Delta t}\mathrm{g}_\eta(\xi,\eta)=\partial[{}^{0}\mathrm{x}(\xi,\eta)+{}^{t+\Delta t}\mathrm{u}_x(\xi,\eta);\ {}^{0}\mathrm{y}(\xi,\eta)+{}^{t+\Delta t}\mathrm{u}_y(\xi,\eta);\ {}^{0}\mathrm{z}(\xi,\eta)+{}^{t+\Delta t}\mathrm{u}_z(\xi,\eta)]/\partial\eta$,
${}^{t+\Delta t}\mathrm{g}_{normal}(\xi,\eta)={}^{t+\Delta t}\mathrm{g}_\xi(\xi,\eta)\times{}^{t+\Delta t}\mathrm{g}_\eta(\xi,\eta)$ with reference to the deformed state of the membrane at $t+\Delta t$.

Constitutive relation between structural stress and structural strain is assumed linear by a constant elasticity tensor that is applied for tensile tangential stress state within the membrane. Consideration of full nonlinearity of stress terms and application of linearized virtual strain terms leads to a consistent approach that takes higher-order nonlinearities into account. The formal expression of analytic force equilibirium for a considered membrane structure in its weak form is

$$\begin{aligned}
&\int\limits_{x_{normal}(\xi,\eta)} \int\limits_{A(\xi,\eta)} \left[S_{\xi\xi}(\xi,\eta)\delta\epsilon_{\xi\xi}(\xi,\eta) + S_{\eta\eta}(\xi,\eta)\delta\epsilon_{\eta\eta}(\xi,\eta) + S_{\xi\eta}(\xi,\eta)\delta\epsilon_{\xi\eta}(\xi,\eta)\right] dx_{normal}(\xi,\eta)dA(\xi,\eta) \\
= &\int\limits_{x_{normal}(\xi,\eta)} \int\limits_{A(\xi,\eta)} \left[\rho\ddot{u}_x(\xi,\eta)\delta u_x(\xi,\eta) + \rho\ddot{u}_y(\xi,\eta)\delta u_y(\xi,\eta) + \rho\ddot{u}_z(\xi,\eta)\delta u_z(\xi,\eta) \right. \\
&\left. + f_x^{ext}(\xi,\eta)\delta u_x(\xi,\eta) + f_y^{ext}(\xi,\eta)\delta u_y(\xi,\eta) + f_z^{ext}(\xi,\eta)\delta u_z(\xi,\eta)\right] dx_{normal}(\xi,\eta)dA(\xi,\eta)
\end{aligned} \tag{1}$$

where membrane stress, virtual membrane strain and domain volume of the membrane can refer either to the undeformed state at 0 or to the deformed state at $t + \Delta t$.

Spatial discretization is performed by 9-node-4-corner finite elements and quadratic polynomials for interpolation of translational displacements within each element [3]. Time discretization is performed by application of the HHT-α method [4].

Regarding the state of force equilibrium it is demanded that the partial derivatives of the virtual work expression in equation (1) with respect to the cartesian components of the unknown displacements $[{}^{t+\Delta t}\mathrm{u}_x(\xi,\eta);\ {}^{t+\Delta t}\mathrm{u}_y(\xi,\eta);\ {}^{t+\Delta t}\mathrm{u}_z(\xi,\eta)]$ are zero for all unknown displacements. Subsequent introduction of spatial discretization and time discretization leads to the discrete nonlinear equation system that gives the appropriate consistent discrete description of the stated problem. The discrete nonlinear equation system can be solved in an iterative manner to eventually obtain the spatially discrete and time discrete unknown displacements $[{}^{t+\Delta t}\mathrm{u}_x^k(\xi,\eta);\ {}^{t+\Delta t}\mathrm{u}_y^k(\xi,\eta);\ {}^{t+\Delta t}\mathrm{u}_z^k(\xi,\eta)]$, $k = 1 \ldots N_k$ (N_k: Number of discrete nodes in the appropriate finite element discretization).

Proper definition of nonlinear membrane strain, linearized virtual membrane strain and the appropriate stiffness and mass contributions in the nonlinear problem together with the analytic membrane area differential are given. Partial derivatives of the nonlinear membrane strain with respect to the unknown displacements are presented. Evaluation of the discrete system for the case of even in-plane load and for the case of out-of-plane load on the unit 9-node-4-corner element is performed. Appropriate stiffness and mass matrix coefficients for the unit 9-node-4-corner element are shown. The presented numeric model is applied to a square Hypar shaped membrane exposed to out-of-plane work load and to a spinnaker sail membrane exposed to horizontal pressure load.

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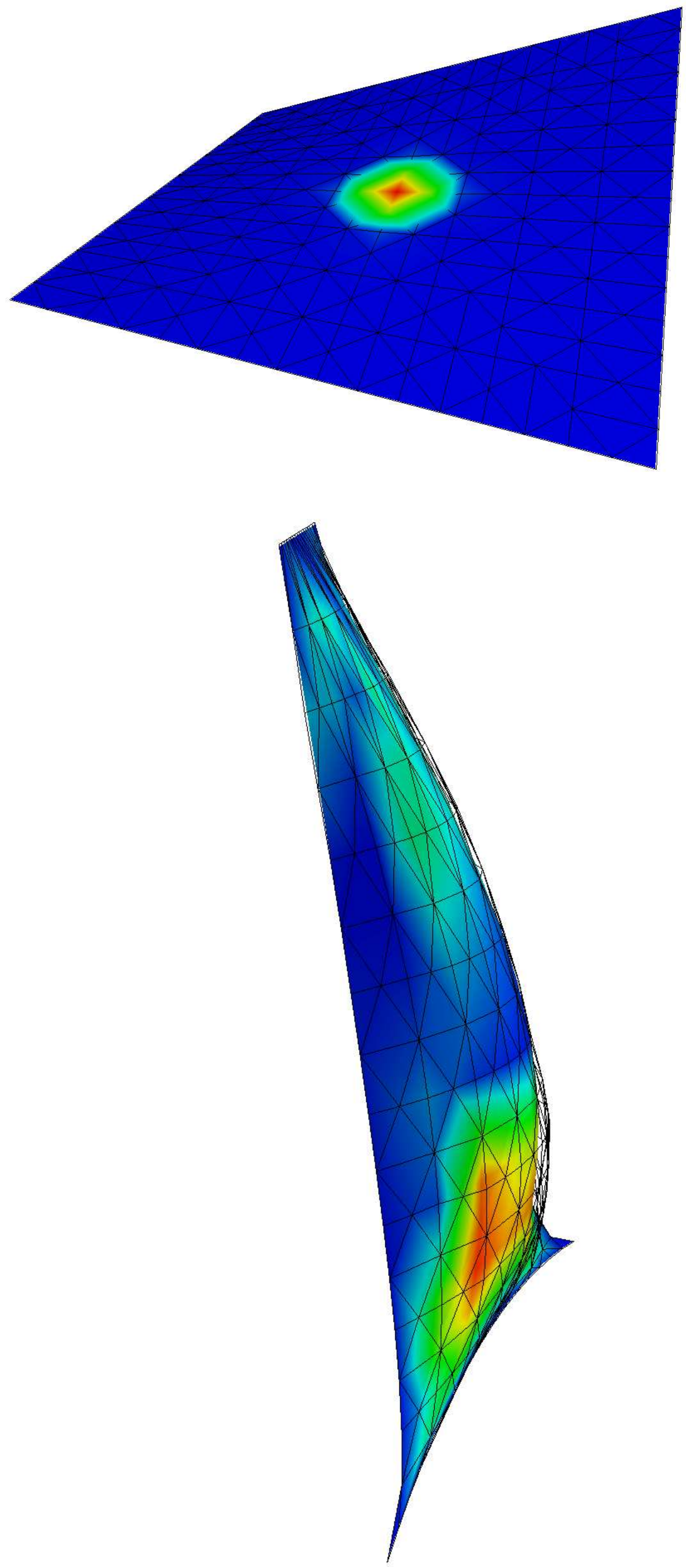

Figure 1: Hypar shaped membrane and spinnaker sail membrane with displacement magnitude (contour) and deformed configuration (wireframe)