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NUMERICAL STUDY OF CONVERGENCE OF THE MASS REDISTRIBUTION METHOD FOR ELASTODYNAMIC CONTACT PROBLEMS

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We consider an elastic bar submitted to impacts with a rigid obstacle (see Figure 1). The mathematical problem is formulated as an elastodynamic contact problem:

$$\rho\ddot{u} - \text{div }\sigma(u) = f \qquad \text{in } \Omega \times (0,T), \tag{1}$$

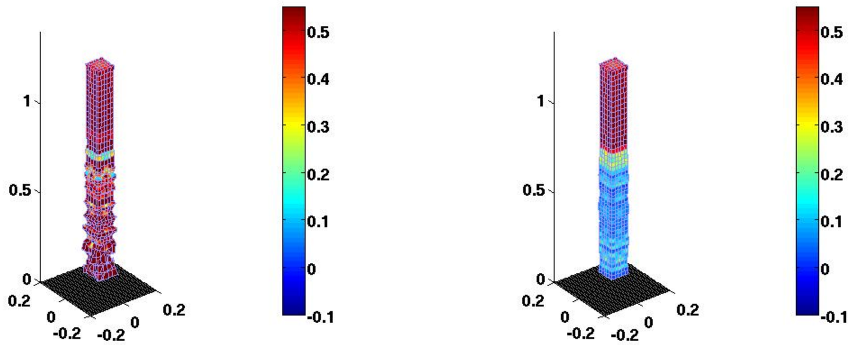
The frictionless Signorini conditions can be written as follows:

$$0 \geq u_\nu \perp \sigma_\nu(u) \leq 0 \quad \text{and} \quad \sigma_\tau(u) = 0 \quad \text{on } \Gamma_C \times (0,T), \tag{2}$$

where ν and $\tau \in \mathbb{R}^d$ are outward normal and tangent unit vectors, respectively, to Ω on $\partial\Omega$. The orthogonality has the natural meaning, namely, an appropriate duality product between two terms of the relation vanishes.

Based on the mass redistribution and the finite elements methods (see [1]), a semi-discretized problem associated to (1)-(2) is presented. The mass redistribution method consists to obtain a modified mass matrix with a vanishing mass on contact nodes. The semi-discretized problem is well-posed and it is energy conserving [1, 2]. Moreover in the one-dimensional case the convergence of the method is established (see [2]). One-dimensional numerical tests with several time-integration schemes illustrate the advantages of mass redistribution method, i.e. in general, the oscillations of displacement are elimitated, the oscillations of contact forces are reduced, the stability of the scheme are recoverd and the convergence rates are improved, see [3].

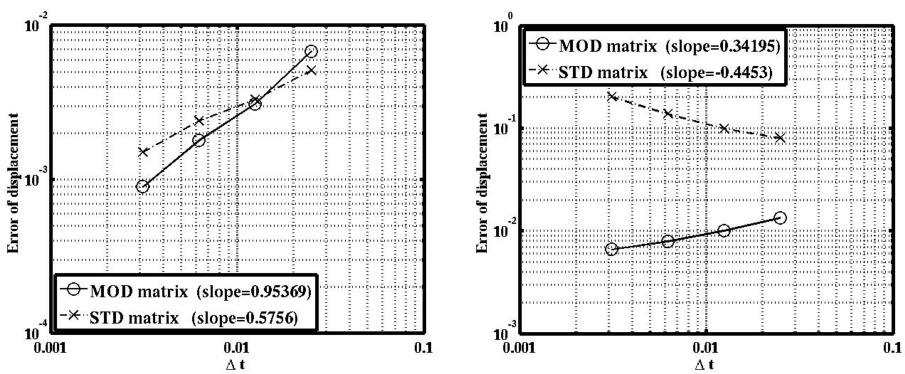
In the present work, we focus on the numerical study of convergence of the mass redistribution method in two and three-dimensional cases. 2D and 3D simulations by using



(a) With standard mass matrix (b) With redistributed mass matrix

Figure 1: Von Mises stress evolution of deformed bar after contact .

the standard and redistributed mass matrices and some time-integration schemes will be presented and analyzed in order to determine the convergence properties of the mass redistribution method (see Figure 2).



(a) $\|U_h^n - u\|_{\mathrm{L}^2(0,T;\mathrm{L}^2(\Omega)}$ (b) $\|U_h^n - u\|_{\mathrm{L}^2(0,T;\mathrm{H}^1(\Omega))}$

Figure 2: Comparison of the convergence curves obtained by using the standard and redistributed mass matrices with Crank-Nicholson scheme.

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