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A MIXED CUBIC ZIGZAG MODEL FOR MULTILAYERED COMPOSITE AND SANDWICH PLATES INCLUDING TRANSVERSE NORMAL DEFORMABILITY

Luigi Iurlaro[1*], Marco Gherlone[1] and Marco Di Sciuva[1]

[1] Politecnico di Torino, Department of Mechanical and Aerospace Engineering,
Corso Duca degli Abruzzi 24, 1029 Torino (Italy)
luigi.iurlaro@polito.it, marco.gherlone@polito.it, marco.disciuva@polito.it, www.aesdo.polito.it

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A reliable design of multilayered composite and sandwich components for primary load-bearing structures of modern civilian and military aircraft requires an accurate evaluation of the stress and strain fields. It is well-known that the Equivalent Single Layer (ESL) models are not suitable for the analysis of multilayered composite and sandwich structures, especially if an accurate local response prediction is required. By contrast, Layer-Wise (LW) approaches commonly ensure accurate global and local response predictions but at the expense of costly computations for laminations composed of a large number of layers. The zigzag class of structural theories, pioneered by Di Sciuva [1], are characterized by an accuracy comparable to that of the LW approaches and a computational cost similar to that of the ESL models.

Recently, Tessler et al. [2] formulated the Refined Zigzag Theory (RZT), a displacement-based structural theory for laminated composite and sandwich plates. Within RZT, a novel set of piece-wise linear functions are added to the in-plane displacements of the First-Order Shear Deformation Theory (FSDT). The transverse displacement is assumed constant along the thickness, as in FSDT, resulting in a seven-kinematic variable RZT model. The RZT accuracy in predicting global and local responses has been already assessed on several applications concerning the elasto-static problem of bending and the eigenvalues problem of free vibration and buckling of multilayered composite and sandwich plates [2,3].

The RZT kinematics assumptions give rise to piece-wise constant through-the-thickness transverse shear stresses, thus violating the continuity requirement at layer interfaces. This drawback has been overcome developing a mixed RZT model ($RZT^{(m)}$) [4], via the Reissner Mixed Variational Theorem, wherein the assumed transverse shear stresses are derived with aid of the three-dimensional equilibrium equations, as proposed by Tessler in [5]. The $RZT^{(m)}$ model shows remarkable accuracy in predicting the actual transverse shear stress field when compared with either the three-dimensional elasticity solutions or those of high-fidelity FEM.

For thick plates, the in-plane displacements tend to exhibit non-linear behavior across the thickness. For this reason, a linear piece-wise model for the analysis of thick multilayered composite and sandwich laminates is not sufficient and a higher-order kinematics assumption has to be adopted. In the framework of the zigzag theories, Di Sciuva [6] was the first to formulate a third-order zigzag plate model able to satisfy *a priori* the stress continuity conditions at layers interfaces. In order to include the thickness stretch effect in relatively thick laminates, Barut et al. [7] enriched the RZT in-plane displacements with a piece-wise quadratic through-the-thickness contribution and a cubic distribution along the thickness of the transverse normal stress is assumed independently.

In this work, a mixed higher-order zigzag model ($CZT^{(m)}$), developed via the Reissner Mixed Variational Theorem, is presented. The in-plane displacements are approximated by adding to the FSDT displacement field a piece-wise cubic through-the-thickness contribution, while the transverse displacement is postulated to vary according to a quadratic polynomial. The assumed transverse shear stresses are derived following the same procedure as in $RZT^{(m)}$, whereas the assumed transverse normal stress varies according to a cubic polynomial along the laminate thickness. The assumed transverse stress tensor components are related with the kinematic variables, thus the $CZT^{(m)}$ results in a nine-kinematic variable model, regardless of the number of layers.
The accuracy of the $CZT^{(m)}$ model is assessed on a series of problems concerning the elasto-static problem of bending and the eigenvalues problems of free vibration and buckling load of multilayered composite and sandwich plates. Comparison with linear and non-linear ESL models and linear zigzag ones (RZT, $RZT^{(m)}$) is made in order to demonstrate the prediction capabilities of the present formulation. The 3D exact Elasticity solution or a high-fidelity FEM one, when the former is not available, is assumed as reference in the comparison. The $CZT^{(m)}$ appears a very promising model for a reliable analysis of thick and highly heterogeneous multilayered composite and sandwich plates.

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