Enhanced visco-elastic FEM analysis of laminated composite plates using a first-order shear deformation theory

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Recently, advanced composite structures have been widely used in various engineering applications due to their light-weight and high-stiffness characteristics. With increasing utilization of laminated composite structures, there are many theories developed to accurately predict their elastic responses, such as the well-known conventional theories (FSDT; first-order shear deformation theory) and many other refined shear deformation theories(EHOPT; efficient higher order plate theory, EFSDT; enhanced first-order shear deformation theory).

Meanwhile, composite structures have visco-elastic characteristics such as creep strain, stress relaxation and time-dependent failure because composite material is composed of elastic fibers and visco-elastic matrix. Thus, visco-elastic effects of the laminated composite structures should be considered for the reliable analysis. This brings us to develop a new type of enhanced first-order shear deformation theory (EFSDT) by employing the concepts of the strain energy relationship and Laplace transformation. This will allow us to analyze accurately the visco-elastic behavior of laminated composite plates under mechanical loading and to investigate visco-elastic effects such as creep strain and stress relaxation.

In this paper, as a way to address the aforementioned issues, the EFSDT in Laplace domain is proposed and applied to the visco-elastic problem. The main objective herein is to systematically modify strain energy of first-order shear deformation theory (FSDT) based on the definition of Reissner-Mindlin's plate theory. To this end, the in-plane warping functions based on the efficient higher-order plate theory (EHOPT) are utilized to improve the performance. The relationships between the FSDT and EHOPT are systematically established in Laplace domain via both the strain energy transformation and the least-square approximation of the mean displacement. And the convolution theorem of Laplace transformation is employed to circumvent the overwhelming complexity of dealing with visco-elastic materials which is expressed as the Bolzmann superposition integral equation. The developed EFSDT has the same computational advantage of the FSDT, while allowing us to improve the local distributions of the visco-elastic responses via the recovery procedure.

To demonstrate the accuracy and efficiency of the proposed theory, visco-elastic behaviors of the cross-ply laminated composite rectangular plates under mechanical loading are analyzed by employing analytical method. And Maxwell and Kelvin models are considered as the



visco-elastic model. The time-dependent normalized in-plane displacements based on the FSDT and EFSDT are presented in Fig. 1. The results in real time domain can be obtained by applying the inverse Laplace transformation. Anti-symmetric cross-ply for the Maxwell model (Fig. 1-a) and symmetric cross-ply for the Kelvin model (Fig. 1-b) are considered. From the numerical examples, one can see that the proposed theory provides accurate results by capturing severe zig-zag variation of in-plane displacements while it requires the same computational cost as compared with FSDT. Numerical results show that the visco-elastic responses for the Maxwell and Kelvin models can be accurately analyzed by utilizing the Laplace transformation.

In a full-length paper, finite element implementation will be performed, and the results of the present theory will be compared to those reported in the open literature. The process of strain energy transformation in Laplace domain and finite element formulation will be described in detail.

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