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A LOCAL OPTIMAL CONTACT CONDITION IN 2D AND 3D

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Finite element methods are currently used to approximate the unilateral contact problems (see, *e.g.*, [5, 7, 8, 9, 10]). Such problems show a nonlinear boundary condition, which roughly speaking requires that (a component of) the solution u is nonpositive (or equivalently nonnegative) on (a part of) the boundary of the domain Ω. This nonlinearity leads to a weak formulation written as a variational inequality which admits a unique solution and the regularity of the solution shows limitations whatever the regularity of the data is. A consequence is that only finite element methods of order one and of order two are of interest. In this presentation, we consider the studies with the finite element methods of order one and two in two and three space dimensions.

In this work (which corresponds to the thesis [4]) we are not only interested in contact problems of a body with a rigid foundation but also in the contact configurations of two bodies whose respective meshes may not coincide on the contact interface. In the latter case which has been considered from a numerical point of view since the 80's, it is now known that the local node-on-segment contact conditions are not satisfactory (in the general case) in comparison with more global approaches inspired from the mortar domain decomposition method [3] and adapted to contact problems (see the early theoretical and numerical works in the late 90's [1, 2, 6]). From a numerical point of view, the mortar concept has been extended to many contact configurations such as friction, quadratic finite elements, large deformations, three dimensional problems...

In this study, we consider a discrete contact condition which differs from the original mortar approach and which requires, in the simplest case of a sole body in contact with a rigid foundation, that the approximate solution u^h is nonpositive in average on some conveniently chosen local patches (comprising one or several contact elements) that form a partition of the contact zone. This very simple local approach can be easily extended to multi body contact with nonmatching meshes in two and three space dimensions. Theoretical results show that this method is as efficient as the original mortar approach and numerical experiments will be shown in the presentation.

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