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DISPERSIVE PROPERTIES OF DPG METHODS FOR ACOUSTICS

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The discontinuous Petrov Galerkin (DPG) method, introduced in [1, 2] finds its numerical solution by minimizing a residual in a nonstandard dual norm. Although dual norms in standard weak formulations are global and difficult to approximate, within the DPG setting, they are local and easily approximable. The DPG methodology immediately found applications in various areas.

The focus of this talk is on its application to wave propagation, specifically the Helmholtz equation. This application was first analyzed in [3] which proved wavenumber-independent error estimates. Although the error estimates indicated the presence of pollution effects, numerical results in [3] could not quantify these effects as it was negligible for the polynomial orders considered there. More extensive numerical studies in [4] confirmed the presence of pollution effects.

After introducing the required background, this talk will be devoted to a presentation of results in [4]. It studies a DPG method where the test space is normed by a modified graph norm. The modification scales one of the terms in the graph norm by an arbitrary positive scaling parameter ε. The main finding is that as the parameter ε approaches zero, better results are obtained (under some circumstances) for the Helmholtz equation. The main tool used for the study is a dispersion analysis on the multiple interacting stencils that form the DPG method. The analysis shows that the discrete wave numbers of the method are complex, explaining the numerically observed artificial dissipation in the computed wave approximations. Comparisons with the discrete wave speeds of other standard methods will also be presented.

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