11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
July 20 - 25, 2014, Barcelona, Spain

A RAPIDLY CONVERGENT ALGORITHM FOR THE SOLUTION OF NAVIER-STOKES EQUATIONS

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Key Words: *finite volume method, unstructured grid, pressure-velocity coupling, segregated method, method of loops*

During last two decades research efforts within the field of CFD has been directed to solving physically and computationally more demanding problems, leaving the Finite volume methods for solving fluid flow problems mostly based on SIMPLE-like methods. SIMPLE algorithm was originally defined on staggered grids [1] on which it is consistent and physically clear. On collocated grids SIMPLE-like methods require non-physical pressure boundary conditions and use of arbitrary interpolations [2] with a possible negative consequence to the solution accuracy. Their convergence rate strongly relies on method's underrelaxation factors. The theoretical research here presented has led to a new pressure-velocity coupling method – FLOP algorithm (Flux LOoping for Pressure drop) which does not suffer from such drawbacks on collocated grids, nor requires underrelaxation factors and that has a significantly higher convergence rate than SIMPLE-like methods, while keeping the segregated solving procedure.

Mathematical model: Incompressible fluid flow is governed by the continuity and momentum equations:

$$\frac{\partial v_i}{\partial x_i}=0 \quad , \tag{1}$$

$$\frac{\partial \rho v_i}{\partial t}+\frac{\partial \rho v_j v_i}{\partial x_i}=-\frac{\partial p}{\partial x_i}+\frac{\partial}{\partial x_j}\left(\mu\frac{\partial v_i}{\partial x_j}\right) \tag{2}$$

here, written with denotations: t – time, x_i – Cartesian coordinate, ρ and μ – constant fluid density and dynamic viscosity, v_i – velocity field and p – pressure field

FLOP algorithm: Following the idea of the original formulation of the SIMPLE algorithm on staggered grids, the new algorithm on collocated grids uses mass fluxes *F* trough the finite volume's faces as main variables (Fig. 1). When mass fluxes satisfy the continuity equation, the velocities at the volume's centres may be obtained by interpolation:

$$v_i=\frac{1}{\rho\Delta V}\sum_{k=1}^{nb} x_i^k F^k \quad , \tag{3}$$

where *nb* designates the number of volume's faces, while x_i^k designates the vectored distance from the volume's centre to the respective face centre

Once the F and v_i are known, the pressure gradient at volume's centres may be calculated from the discretized form of equation (2).

By applying (3) to (2) for a single finite volume with centre A, this is formally written as:

$$\left.\frac{\partial p}{\partial x_i}\right|_A = f_i\left(F^k\right) \quad , \; k = 1, 2, \ldots, nb_A. \qquad (4)$$

From all possible solutions of (1) for mass fluxes, we search for the one that defines a unique pressure field. This condition is defined as zero pressure drop (Δp_L=0) for (4) integrated along closed loops. Next, Δp_L-s = 0 are expressed as functions of ΔF_L-s – the unknown corrections of fluxes required to achieve them. The resulting system is solved by a Newton's method.

As depicted in Fig.1, the ΔF_L-s are such assembled that continuity equation remains satisfied after each correction.

By solving the Laplace's equation, the algorithm's procedure starts from mass fluxes F which satisfy the continuity equation. Thereafter, the algorithm iterates solely on the solution for corrections ΔF_L-s, to obtain a zero (sufficiently small) Δp_L in all loops, which then is - the solution of the problem. Finally, the actual pressure field is reconstructed from such obtained pressure gradient field.

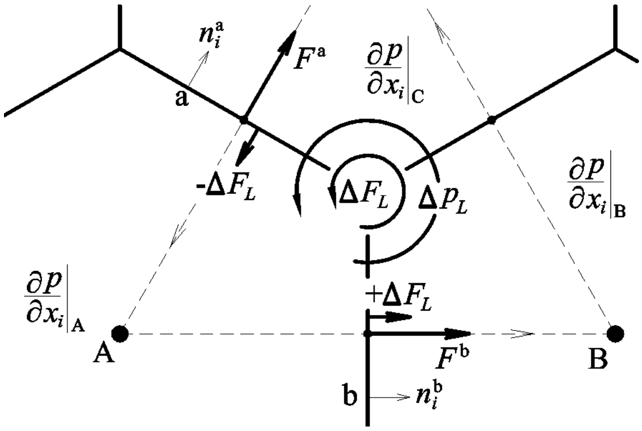

Figure 1. FLOP algorithm variables assembly

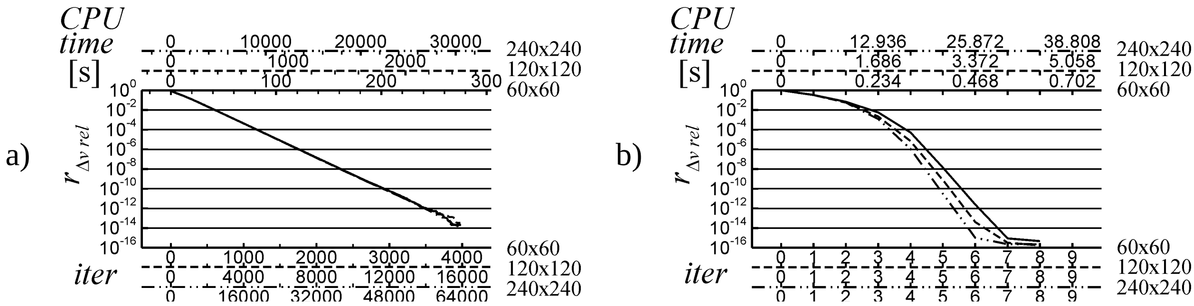

Figure 2. 2D lid-driven cavity laminar flow at Re = 100, solved for three different grid sizes. The convergence histories for a) SIMPLE algorithm, b) FLOP algorithm.

Concluding remarks: The new FLOP algorithm implemented in own computer program was compared with the SIMPLE algorithm. In all performed tests, the algorithms behave similarly to the case depicted in Fig. 2. The FLOP algorithm shows a significantly higher convergence rate and efficiency, as well as insensitivity of No. of iterations to the grid size.

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