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HIGH ORDER AUTOMATIC DIFFERENTIATION OF CONTITUTIVE LAWS AND APPLICATION TO PLASTIC STRUCTURES

A. Lejeune[1], H. Boudaoud[2], N. Mathieu[2] and M. Potier-Ferry[2]

[1]Institut FEMTO-ST, Université de Franche-Comté, F-25000 Besancon, France,
arnaud.lejeune@univ-fcomte.fr

[2] LEM3, Université de Lorraine, F-57045 Metz, France,
hakim.boudaoud@univ-lorraine.fr, norman.mathieu@univ-lorraine.fr, michel.potier-ferry@univ-lorraine.fr

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The Asymptotic Numerical Method is a path following technique, where the prediction relies on high order Taylor series and where corrections steps are generally not necessary. The use of Taylor series is a very efficient technique to manage automatically the continuation procedure, what is useful in cases of non-linear response curves, for instance in the presence of bifurcation. This has been applied to many cases, especially in nonlinear elasticity, fluid mechanics [4] and plasticity [1]. In this talk, a new procedure is presented to implement more easily various plastic constitutive laws.

The numerical technique is based on the computation of high order derivatives of response curves in plasticity, i.e. high order derivatives of plastic constitutive laws and high order derivatives of the corresponding structural problems. Of course the transitions between elastic unloading and plastic loading lead to discontinuous tangent moduli and it is necessary to regularize the constitutive laws to be able to compute Taylor series within plasticity. In [1], regularization methods have been proposed that do not affect the effectiveness of structural calculations and we adopt them in this talk.

Often plastic laws are rather intricate, many physical models can be considered and algorithms to compute Taylor series are not simple. Therefore implementation of this procedure should be user friendly and this can be achieved by using Automatic Differentiation (AD) techniques [2]. That is the goal of this paper. AD could be viewed as a way to generate a code that computes the derivatives of a function, but only the function itself has to be implemented by the user. A natural approach is the so called “forward mode” based on operator overloading. In [3], the same idea had been applied to two algebraic laws: nonlinear elasticity and unilateral contact with tools of any form. Here a generic procedure for Differential Algebraic Equations (DAE) is discussed and applied to small strain and large strain plasticity, but it could be naturally extended to friction.

The derivatives of a plastic stress-strain law lead to an affine relation between the Taylor coefficients of stress and strain, as follows [1, 4]

$$\boldsymbol{\sigma}_k = \mathbf{L}_t : \varepsilon_k + \boldsymbol{\sigma}_k^{nl}$$

where $\boldsymbol{\sigma}_k^{nl}$ depends on the quantities calculated at previous orders and $\mathbf{L}_t$ is the tangent matrix. The computation of this quantity $\boldsymbol{\sigma}_k^{nl}$ is the key point and it will be done via a standard procedure of Automatic Differentiation that can be applied to any constitutive law in a DAE form, such as plasticity, viscoplasticity or friction.

The remainder of the procedure is rather classical within Asymptotic Numerical Method: the application to structural problems is straightforward because it is easy to calculate derivatives of balance laws, the step length is defined a posteriori by evaluating the radius of convergence and new step lengths are computed in the same way. Numerical applications to plastic structures will be presented.

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