11th. World Congress on Computational Mechanics (WCCM XI)
5th. European Conference on Computational Mechanics (ECCM V)
6th. European Conference on Computational Fluid Dynamics (ECFD VI)
July 20 - 25, 2014, Barcelona, Spain

OGDEN PARAMETER OPTIMIZATION FOR FINITE ELEMENT MODELLING OF CERVICAL LIGAMENTS USING HYBRID FORMULATION

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Key words: *Cervical spine, ligaments, FE modelling, hybrid formulation, Ogden hyperelasticity, material parameter optimization, model calibration.*

The constitutive behaviour of the ligamentous tissue has a great influence on the overall response of the cervical spine in finite element modelling. An optimization program has been developed in Matlab to calibrate the material parameters of the ligaments using a least-squares method. The program is coupled with the in-house non-lineal FE code PLCd, which is used to numerically calculate the rotation-moment curves that must be fitted to the published experimental corridors [1, 2, 3] via the ligament parameter identification.

A finite element model of a functional spinal unit (FSU) of the human cervical spine is used for the fitting. The geometry of the vertebrae and discs of the cervical spine was obtained from BodyParts3D 🅭 and the main ligaments providing stability to the FSU were added to complete the model.

The non-lineal response exhibited by ligaments is modelled with an isotropic quasi-incompressible Ogden hyperelastic constitutive model:

$$\mathbf{S} = \sum_{A=1}^{3} \beta_A \mathbf{M}_A - pJ\mathbf{C}^{-1} \quad ; \qquad \beta_A = \sum_{i=1}^{3} \mu_i \left(\tilde{\lambda}_A^{\alpha_i} - \frac{1}{3} \sum_{p=1}^{3} \tilde{\lambda}_p^{\alpha_i} \right) \tag{1}$$

Neo-Hookean hyperelasticity is used to represent the constitutive behaviour of vertebrae and intervertebral discs :

$$\mathbf{S} = \mu J^{-2/3} \left(\mathbf{I} - \frac{1}{3} I_{\mathbf{C}} \mathbf{C}^{-1} \right) - pJ\mathbf{C}^{-1} \tag{2}$$

These quasi-incompressible hyperelastic constitutive laws require the use of hybrid u/p formulation since standard displacement-based FE formulations result in an ill-conditioned stiffness matrix and locking problems.The formulation proposed by Crisfield [4] has been implemented in the FE code. Static condensation at element level eliminates the pressure degrees of freedom, resulting in the following condensed equation of motion:

$$\underbrace{\left(\mathbf{K}_{uu} - \mathbf{K}_{up}\mathbf{K}_{pp}^{-1}\mathbf{K}_{up}^{T}\right)}_{\overline{\mathbf{K}}}\hat{\mathbf{u}} = \mathbb{F}^{e} - \underbrace{\left(\mathbb{F}_{u}^{int} - \mathbf{K}_{up}\mathbf{K}_{pp}^{-1}\mathbb{F}_{p}^{int}\right)}_{\overline{\mathbb{F}^{int}}} \tag{3}$$

All components of the model were meshed using 10-noded quadratic tetrahedral elements with a single pressure point. The material properties where extracted from literature, including the starting value of the parameters to be calibrated. The FSU is subjected to various moments for each iteration of the optimizer in order to obtain the Ogden material parameters which result in a better adjustment of the results to the experimental corridors. The FEM problem is solved using non-lineal large deformation theory in a total Lagrangian framework.

The optimizer is capable of identifying the material parameters of the ligaments such that the FE results are in close agreement to the corresponding experimental corridors. The material restrictions imposed on the optimization parameters ensure the convergence of the FE solutions. However, the fact that, for each parameter to be calibrated, the optimizer must launch a full FE calculation of each desired moment is a considerable computational drawback. Therefore, it has been necessary to perform a sensitivity analysis of the six Ogden parameters of each of the five ligaments modelled to select the parameters which influence the most each response and, thus, reduce the quantity of parameters to optimize.

In conclusion, the optimization program provides a novel automatic way of identifying the material properties of specimen-specific FE models of cervical spines. However, further study into the restrictions imposed on the optimization parameters and the reduction of the computational cost will be required in order to improve the convergence rate of the method and make it useful from a bio-medical point of view.

REFERENCES

[1] J.A. Wheeldon, F.A. Pintar, S. Knowles and N. Yoganandan. Experimental flexion/extension data corridors for validation of finite element models of the young, normal cervical spine. *J. Biomech.*, Vol. **39**, 375–380, 2006.

[2] N. Yoganandan, F.A. Pintar, B.D. Stemper, C.E. Wolfla, B.S. Shender and G. Paskoff. Level-Dependent Coronal and Axial Moment-Rotation Corridors of Degeneration-Free Cervical Spines in Lateral Flexion. *J. Bone Joint Surg. Am.*, Vol. **89**, 1066–1074, 2007.

[3] N. Yoganandan, B.D. Stemper, F.A. Pintar, J.L. Baisden, B.S. Shender and G. Paskoff. Normative Segment-Specific Axial and Coronal Angulation Corridors of Subaxial Cervical Column in Axial Rotation. *Spine*, Vol. **33**, 490–496, 2008.

[4] Crisfield. Non-linear finite element analysis of structures (Vol 2), Wiley, 1991.