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Crease-type solutions in a coated elastic half-space

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Linear buckling analysis for a homogeneous half-space was first carried out by Biot in 1956 [1], but its post-buckling behaviour has eluded full understanding for many decades. The difficulty lies in the fact that a homogeneous half-space does not have a natural lengthscale and as a result there does not exist a distinguished mode number -- all mode numbers having equal status. A naive weakly nonlinear postbuckling analysis would be to write the solution as a Fourier integral or Fourier series and impose a solvability condition as the second order of successive approximations. This procedure would result in an infinite system of quadratic equations which do not seem to have convergent non-trivial solutions. This indicates that if postbuckling solutions existed, they should contain some form of discontinuities, e.g. in the form of static shocks. The next natural step is then to approach such non-smooth solutions by a limiting process. One such scheme is to assume that the surface of the half-space is corrugated and to follow the evolution of surface profile as lateral compression is increased gradually. It was shown by Fu [2] that the evolution is indeed terminated by the formation of a static shock wave, and that the bifurcation is likely to be subcritical. A similar scheme was employed by Cao and Hutchinson [3] that was also augmented by a full numerical simulation. Another scheme is to first assume that the half-space is coated by a thin layer with bending stiffness and then let the bending stiffness tend to zero. This scheme was carried out by Hohlfeld and Mahadevan [4] where analysis of a simple model was combined with a full numerical simulation.

We study the post-buckling behaviour of a coated elastic half-space in which the coating and half-space have very similar material properties. This can be viewed as another scheme to “unfold" the buckling problem associated with an un-coated homogeneous elastic half-space.
Our assumption about the material properties enables us to bring in all Fourier modes in a weakly nonlinear analysis in a self-consistent manner. The problem is eventually reduced to the solution of an infinite system of quadratic equations. It is shown that our weakly nonlinear analysis can reproduce a number of fully nonlinear phenomena reported previously in the literature, including sub-critical crease-like solutions.

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