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FINITE ELEMENT ERROR ESTIMATES ON THE BOUNDARY FOR ELLIPTIC BOUNDARY VALUE PROBLEMS WITH NEUMANN BOUNDARY DATA

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This talk is concerned with discretization error estimates for the boundary value problem

$$\begin{aligned} -\Delta y + y &= f \qquad \text{in } \Omega, \\ \partial_n y &= g \qquad \text{on } \partial\Omega, \end{aligned}$$

where the data are smooth and the domain Ω is assumed to be polygonal but not necessarily convex. The partial differential equation is discretized by linear finite elements on quasi-uniform meshes. We focus on the derivation of finite element error estimates in the L^2-norm on the boundary. For example such estimates are required in the context of Neumann boundary control problems. Using common approaches known from the numerical analysis for pde's, such as the Aubin-Nitsche method for estimates in $L^2(\partial\Omega)$, one can prove a convergence order of $\max(3/2, 1/2 + \pi/\omega - \epsilon)$ where ω denotes the largest interior angle of the domain and ϵ an arbitrarily small constant. This rate is sharp if the domain is non-convex. In convex domains one can first apply the embedding $L^p(\partial\Omega) \hookrightarrow L^2(\partial\Omega)$ with some $p \in (2, 2/(2-\pi/\omega))$ and afterwards the Aubin-Nitsche method for estimates in $L^p(\partial\Omega)$ to deduce an approximation rate of $2 - 1/p$. Thus, one can expect a convergence order close to 2 if the largest interior angle ω is less than $\pi/2$. Despite of this, numerical experiments indicate that this theoretical approximation rate can still be improved. In this talk we show that a convergence order of $O(h^2|\ln h|^{3/2})$ can be obtained up to an interior angle of $2\pi/3$ by using regularity results in weighted Sobolev spaces. For larger interior angles we get the rate $1/2 + \pi/\omega - \epsilon$ due the presence of corner singularities. In that case we use appropriately graded meshes to compensate this lowering effect. All the theoretical findings are illustrated by numerical experiments. Finally, we outline how the results can be extended to a specific class of semilinear elliptic equations. This is joint work with Thomas Apel and Arnd Rösch.

REFERENCES

[1] Th. Apel and J. Pfefferer and A. Rösch. Finite element error estimates on the boundary with application to optimal control. *DFG Priority Program 1253*, Preprint SPP1253-136, 2012. Accepted by Math. Comp.