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COMPARISON OF HIGHER ORDER METHODS IN TIME AND SPACE FOR THE NUMERICAL SIMULATION OF ULTRASONIC WAVE PROPAGATION

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Nondestructive monitoring of structural degradation is often performed by measuring the propagation of ultrasonic waves. The application of these methods ranges from small scale experimental test specimens to large scale structural components. The ultrasonic velocity is a material property that is relatively simple to measure. For a linear elastic and isotropic material, it depends on the the Young's modulus, the density and the Poisson ratio. In experiments it is observed that material degradation is accompanied with a reduction in the ultrasonic velocity [1]. The challenge for the application to real problems is the establishment of a correlation between the reduced velocity and material properties of the specimen. This is even more important in cases with spatially localized degradation such as cracks. Another important influence are the external conditions such as the moisture content. An accurate interpretation of experimental results requires an adequate numerical model that comprises all the relevant features such as e.g. mechanical degradation, temperature or moisture content.

Wave propagation is often modeled using the finite difference approach. Although attractive due to its simple implementation, the modeling of complex geometries is rather difficult. An alternative is the finite element method that has been extensively used to model dynamic characteristics of structural components. However, the accuracy of the method is strongly related to the mesh size [2], and especially for higher frequencies the requirement of 10 to 20 nodes per wave length often becomes prohibitively expensive, especially for 2D and 3D.

The spectral element method, originally developed by [3] for the simulation of laminar flow, is used for the simulation of ultrasonic wave propagation. It is essentially a finite element method with sub-parametric elements showing spectral convergence. The nodes in the natural coordinate system are located at the integration points of a Gauss-Lobatto quadrature. As a consequence, an additional mass lumping scheme to obtain a diagonal mass matrix is not required. The extension to higher dimensions is performed using the tensor product of the one-dimensional shape functions. Unfortunately, this approach is not valid for triangles or tetrahedra, which are attractive for meshing of irregular domains. Strict Gauss-Lobatto quadrature rules for a triangle do not exists [4]. In this paper, the accuracy of spectral finite elements is compared to higher order isoparametric finite elements (triangular and quadrilateral with HRZ-mass lumping) for the propagation of elastic waves. Special emphasis is placed on an accurate capturing of the wave speed for the relevant frequencies (50-150kHz).

Besides the spatial discretization, the temporal integration scheme is of fundamental importance. For the wave propagation problem, explicit methods are usually more advantageous. Due to accuracy requirements the time step in implicit schemes is not considerably larger than in explicit schemes, with the additional cost of solving a system of equations. A commonly used integration scheme is the Verlocity-Verlet algorithm, which is symplectic integrator of second order. Using a time step close to the the critical time step, an accurate solution of the wave propagation required a very fine mesh. Higher order schemes showed a considerable improvement in terms of the accuracy of the solution as a function of computational effort. In this context, a Nyström method is used. It is based on a transformation of the semi-discrete (already discretized in space) second order ordinary differential equation to a system of first order equations with some simplifications making it computationally more attractive than standard Runge-Kutta schemes. Here, an approach based on a 4-th order symplectic scheme is used [5]. In a final example, the P-wave speed determined in virtual experiments for different discretization techniques in space and time is compared.

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