

# NEW SIMPLE, CHEAP AND EFFICIENT EXPLICIT RESIDUAL ERROR ESTIMATOR FOR ADAPTIVE FINITE ELEMENT ANALYSIS IN ELASTICITY AND FRACTURE

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A new mathematically sound a posteriori error estimator with upper bound property is presented to assess the accuracy of primal finite element solutions for problems of linear static elasticity and fracture. The estimator resembles the well-known explicit residual-based error estimator of Babuška and Miller, for which an upper bound on a discretization error, measured in the energy norm, is explicitly computable up to an unknown multiplicative constant.

Different to this classical estimator, the proposed new one has four computable constants, which are derived in a closed form for triangular elements, regardless the order of approximating polynomial. This makes our upper error bound indeed computable and, as a result, provides an adequate stopping criterion for (adaptive) refinements w.r.t. a user-assigned error tolerance. The upper bound is guaranteed, requires small (nearly negligible) computational effort and is sharp. The effectivity index is found to be in the range of 1.2–2.0 for a number of regular and singular benchmark problems, including crack propagation.

Thus, in terms of efficiency judged against simplicity and inexpensiveness, the presented explicit error estimator is superior to, e.g., corresponding implicit residual estimators and may be seen as a very competitive one in comparison with other known, yet more complex a posteriori error estimation techniques.

The new estimator in its present form is valid for primal FEM in linear elasticity, using P1-triangular elements; extension to higher order triangles requires additional constants to be derived and accounted for. Also, a similar derivation strategy is straightforwardly applicable to quadrilaterals, as well as to 3D finite elements, e.g. tetrahedrons. Finally, extensions to goal-oriented error estimation, as well as to advanced finite element formulations like, e.g., GFEM/XFEM, are feasible and look promising.

## REFERENCES

[1] T. Gerasimov, E. Stein and P. Wriggers, New simple, cheap and efficient explicit error

estimator for adaptive finite element analysis in elasticity and fracture. *Int. J. Numer. Meth. Engng.*, submitted 2013.