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ON A HIERARCHICAL MODEL REDUCTION ALGORITHM FOR ELASTIC MULTI-STRUCTURES

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The present paper is devoted to construction and investigation of dimensional reduction algorithm for three-dimensional static and dynamical problems for multi-structures. Multi-structures are elastic bodies, which consist of several parts with different geometrical shapes. Various engineering constructions are multi-structures consisting of plates, shells, beams and other substructures and therefore mathematical modeling of them is important from practical as well as theoretical point of view. One of the first theoretical investigations of multi-structures was carried out by P.G. Ciarlet, H. Le Dret, R. Nzengwa [3]. Applying the asymptotic method they constructed and investigated a mathematical model defined on the union of three-dimensional and two-dimensional domains for a multi-structure consisting of three-dimensional body with a plate clamped in it. The eigenvalue problem for this multi-structure was considered by F. Bourquin, P.G. Ciarlet [2]. Multi-structures consisting of plates and rods were considered by H. Le Dret [4]. The methods used in these works consist in scaling of each part of multi-structure independently of the other, passing then to the limit in the variational equations and identifying the limit. Further, many works were devoted to mathematical modeling and numerical solutions for elastic multi-structures (see [5] and references given therein). It should be pointed out that the above-mentioned methods of construction of mathematical models of multi-structures and the corresponding lower dimensional problems can be used only in the case of infinitely small thickness or width of substructures, while the above quantities are never infinitesimally small in practical applications. Different approach for constructing two-dimensional models of elastic plates with variable thickness was suggested by I. Vekua [6], which was based on approximation of the components of the displacement vector-function of plate by partial sums of the orthogonal Fourier-Legendre series with respect to the variable of plate thickness. Note that I. Vekua's hierarchical dimensional reduction method is one of the spectral approximation methods. The classical Kirchhoff-Love and Mindlin-Reissner models can be incorporated into the hierarchy obtained by I. Vekua, and so it can be considered as an extension of the widely used engineering plate models. Later on, various investigations were devoted to study of mathematical models constructed by I. Vekua's dimensional reduction method and its generalizations for elastic plates, shells and rods (see [1] and references given therein).

In this paper applying spectral method we construct and investigate algorithm of approximation of three-dimensional boundary and initial-boundary value problems for elastic multi-structures by a sequence of problems defined on a union of domains with two different dimensions. Note that the constructed approximations of the three-dimensional problems can be treated as hierarchical mathematical models of multi-structures. We consider a multi-structure, which is a junction of three-dimensional body with general geometrical shape and multilayered substructure clamped in it. The multilayered substructure consists of several layers, which are shells with variable thicknesses. Along the interfaces between three-dimensional body and multilayered substructure and between adjacent shells rigid contact conditions, i.e. continuity of the displacement and stress vector-functions, are given. The three-dimensional models of shells we approximate by two-dimensional ones and construct static and dynamical hierarchical models of multi-structure defined on the union of three-dimensional and several two-dimensional domains. We investigate the constructed hierarchical model reduction algorithm for static and dynamical models of multi-structure. More precisely, we prove the existence and uniqueness of solutions of the boundary and initial-boundary value problems corresponding to the constructed hierarchical models in suitable Sobolev spaces. Moreover, we prove the convergence of the sequence of vector-functions of three space variables restored from the solutions of the obtained pluri-dimensional problems to the solution of the original three-dimensional problem in corresponding spaces and if it satisfies additional regularity conditions we estimate the rate of convergence.

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