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MULTI-SCALE ELASTICITY IDENTIFICATION USING MODIFIED CONSTITUTIVE RELATION ERROR

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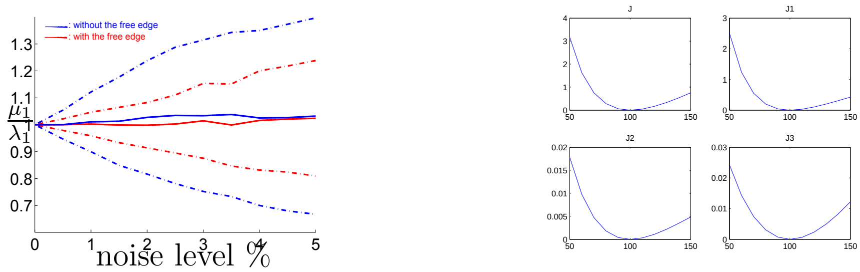
The development of full field kinematic measurements can offer a rich 2D information allowing to take into consideration heterogeneous tests. A key point is then to develop or adapt identification strategies to such measurements. Some specific methods have been proposed in the past years and a review can be found in [1]. Most of these methods consider the identification problem only on the measured area and would usually need boundary conditions to be performed. However, the boundary condition are not always completely known. Even if they are known (e.g. free edge), they might not be on the boundary of the measurement zone. There is hence a need of identification method allowing both the identification over the whole specimen, and the dealing of missing boundary condition. **The modified constitutive relation error (M-CRE)** [2] as proposed here is a good method to identify static elastic properties, which allows to deal with such situations through the taking into account of the whole available information from a theoretical and experimental point of view [3]. Furthermore, considering the identification of heterogeneous elastic properties, the lack of information outside the measurement zone prevents from identifying heterogeneous properties in this area, hence we propose a two-scale approach where heterogeneous properties are sought at the measurement level and homogeneous ones at the specimen/structure level. The M-CRE is a suitable framework to achieve such multi-scale identification.

The principle of the modified constitutive relation error is to define the mechanical fields and material properties as a trade-off between all the available information. The various equations verified by the stress $\underline{\underline{\sigma}}$ and the displacement $\underline{u}$ of the identification problem are split into two groups: (a)The group of reliable equations, which should be exactly verified, with the equilibrium equation, the kinematic compatibility and the free edge assumption; (b)The group of less reliable equations, which is only verified at best by minimizing a functional, with the constitutive relation (parameters $\underline{\theta}$ are sought), the equality to the displacement measurements $\underline{\tilde{u}}$. Depending on the case, the load measurements can be either in group (a) or (b). Then, $(\underline{\underline{\sigma}}, \underline{u}, \underline{\theta})$ are sought as the solution of a minimization



(a) The effect of noise (b) The objective function

Figure 1: Identification results on the examples

problem. In practice, the minimization is split into a sequential minimization: (1)For a given $\underline{\theta}$, find $(\underline{u}(\underline{\theta}), \underline{\sigma}(\underline{\theta}))$ minimizing the objective function J under the constraints of equation group (a). A finite element formulation of the basic problem is proposed and solved based on a QR algorithm. (2)Defining: $\mathcal{G}(\underline{\theta}) = J(\underline{u}(\underline{\theta}), \underline{\underline{\sigma}}(\underline{\theta}), \underline{\theta})$, the identification of $\underline{\theta}$ is performed as the minimization of $\mathcal{G}(\underline{\theta})$. For the multi-scale identification, stress and displacement are defined at both the micro and the macro scales. Then, a coupling scheme is to be defined between the scale, leading to a basic problem defined at the two scales with the corresponding coupling equations. A first coupling is proposed based on the operators from [4] and other formulations are to be studied.

Results concern 2D numerical applications for the mono-scale approach. Figure 1a presents the identification of heterogeneous properties of a plate with an inclusion with results of mean value and standard deviation of the identified $\frac{\mu_1}{\lambda_1}$ as a function of the relative noise level. The taking into account of free-edge improved significantly the identification result-s. On the multi-scale, we took a homogeneous bar as a first analysis. The cost function is presented in figure 1b leading to an accurate identification the Young's modulus. Further applications are under progress, based on axisymmetric structure and 2D plate.

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