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INVESTIGATION OF A FINITE ELEMENT SOLUTION FOR A MECHANICALLY STIMULATED BIOCHEMICAL FRACTURE HEALING MODEL

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Following the fracture of a bone, a well orchestrated cascade of cellular events usually leads to the reunion of the fractured bone ends and to the recovery of its original functionality. Suitable biochemical and mechanical conditions within the fracture region are required for a successful regeneration. Recent computational approaches attempt to combine biochemical and biomechanical stimuli. The simulation of the chemical events involve the concurrent solution of several non-linear hyperbolic differential equations. The arising stability issues within the finite element framework will be discussed and a stabilization scheme, utilizing the time-discontinuous Galerkin (TDG) and the Finite Calculus (FIC) methods, will be presented. The impact of mechanical stimulation on the simulated healing process will be also investigated.

The coupled partial differential model applied in this context was developed originally by Geris et al. [1]. It consists of twelve equations which represent the evolution and activities of five cell types (mesenchymal stem cells, fibroblasts, chondrocytes, osteoblasts and endothelial cells), four tissues (fibrous tissue, cartilage, woven bone and vasculature) and three types of growth factors (chondrogenic, osteogenic and angiogenic growth factors). In general these equations contain advective, diffusive and/or reactive terms, where the coefficients depend in part on the concentrations of the unknowns.

Therefore, the underlying mathematical problem for the biochemical part of the simulation can be stated as follows, solve

$$\frac{\partial \phi}{\partial t} = -\frac{\partial}{\partial x}(a(x,t)\phi) + \frac{\partial}{\partial x}(d(x,t)\frac{\partial \phi}{\partial x}) + r(x,t)\phi, \tag{1}$$

for $\phi(x,t)$ in a domain Ω, fulfilling the inital condition $\phi(x,0) = \phi_0$ in Ω and suitable boundary conditions on $\partial\Omega$. With a, d, and r being the time dependent advection, diffusion and reaction coefficients, respectively.

It is well known, that spurious oscillations are likely to pollute the numerical solution of this problem, while further stability issues arise from the time integration. In this work, the necessary stabilization was achieved by combining the Finite Calculus Method, originally introduced by Oñate et al. [2] and the Time-Discontinuous Galerkin method.

The biochemical simulation of the fracture healing process is furthermore coupled with a simulation of the mechanical excitation of the callus region. Appropriate measures for the local mechanical demand are investigated and will define a biomechanical stimulation acting on the different cell types. In this work the stem cell differentiation will be influenced by the calculated stimulus and tissue formation will proceed accordingly. Measures for the mechanical demand include the second invariant of the Green-Lagrange strain tensor, given by the principal stretch ratios λ_i

$$\varepsilon_V = \frac{\sqrt{2}}{6}\sqrt{(\lambda_1-\lambda_2)^2+(\lambda_2-\lambda_3)^2+(\lambda_3-\lambda_1)^2}. \tag{2}$$

and/or the local hydrostatic pressure. Results obtained by this model for a two dimensional, axisymmetric callus domain will be presented (see e.g. 1) and discussed.

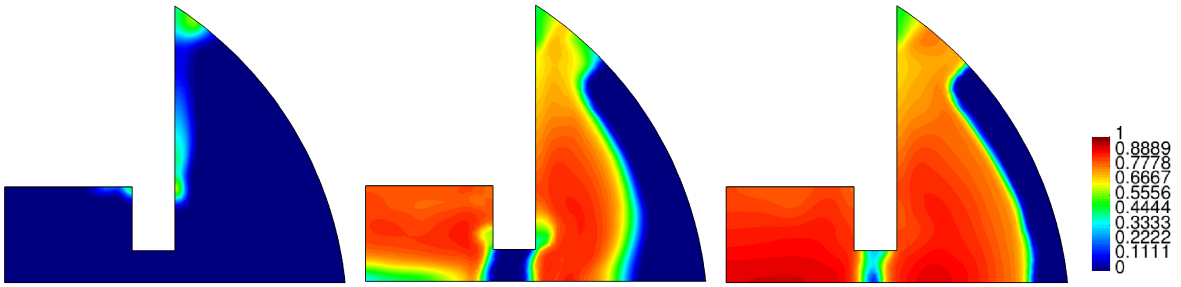

Figure 1: Immature bone density distribution inside the callus region, at day 10, 20 and 30 post fracture (from left to right).

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