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EIGENVALUE ANALYSIS OF AN AXIALLY MOVING STRING WITH MULTIPLE ATTACHED OSCILLATORS USING GREEN'S FUNCTION METHOD

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So far there have been extensive studies on a variety of axially moving continua. However, the eigenvalue problem for moving systems with multiple attached oscillators has not been fully resolved in spite of several publications regarding vibratory systems with moving masses. For moving systems subjected to complex constraints, its dynamic characteristics can be significantly influenced by the coupling effect between main system and the attached oscillators. For example, for moving oscillator systems the presence of mass-spring oscillator varies the inertia of the system, which results in location-dependent eigenvalues that are different from conventional analytical eigenvalue solutions. The present paper deals with the eigenvalue problem for the axially moving string with multiple attached mass-spring oscillators. We consider a simply supported, axially moving string with a velocity c between two fixed eyelets with N attached undamped linear oscillators. Assume the ith oscillator arrives at the left end of the string at time $t_i, i=1,\cdots,N$, of which the coefficient of acting time is $\hbar_i = H(t-t_i) - H(t-(t_i+1/c))$, where $H(\cdot)$ is the Heaviside unit-step function. By neglecting the flexural, shear and torsion rigidity, the nondimensional governing equations of transverse vibration of the system can be written in the form

$$w_{,tt} + \mu\left(w_{,t} + cw_{,x}\right) + 2cw_{,xt} + \left(c^2-1\right)w_{,xx} = \sum_{i=1}^{N} k_i\hbar_i\left(y_i - w\right)\delta\left(x-x_i\right) + f_w \tag{1}$$

$$\ddot{y}_i + \frac{k_i}{m_i}\left(y_i - w\left(x_i,t\right)\right) = f_i,\ t_i \le t \le t_i + 1/c,\ i = 1,\cdots,N \tag{2}$$

where $w(x,t)$ is the transverse displacement of the string, μ is the modal damping of the string. y_i, k_i and m_i are the deflection, the stiffness coefficient and mass of the ith oscillator, respectively. f_w and f_i are external force on the string and mass, respectively. $\delta(\cdot)$ is the Dirac-Delta function. Let the general solutions of the foregoing be

$$w(x,t) = \mathrm{Re}\left\{\phi(x)e^{\lambda t}\right\} \text{ and } y_i(t) = Y_i e^{\lambda t} \tag{3}$$

By applying the Green's function method, the solution of Eq. (4) can be obtained，

$$\phi(x) = -\sum_{i=1}^{N}\varepsilon_i\hbar_i\phi\left(x_i\right)G\left(x;x_i\right)$$

where $\varepsilon_i = m_i k_i\lambda^2/(m_i\lambda^2+k_i)$ and $G(x;\xi)$ is the Green's function satisfying

$$\left(\lambda^2+\mu\lambda\right)G\left(x;\xi\right) + \left(2c\lambda+\mu c\right)G'\left(x;\xi\right) + \left(c^2-1\right)G''\left(x;\xi\right) = \delta\left(x-\xi\right) \tag{4}$$

and is with boundary condition

$$G(0;\xi) = G(1;\xi) = 0\,. \tag{5}$$

A set of N homogeneous equations can be derived for the modal functions:

$$\phi(x_i) + \sum_{j=1}^{N} \varepsilon_j \hbar_j \phi(x_j) G(x_i; x_j) = 0,\; i = 1,\cdots,N \tag{6}$$

The natural frequency of the system can be obtained by solving the following equation:

$$\begin{vmatrix} \varepsilon_1\hbar_1 G(x_1;x_1)+1 & \varepsilon_2\hbar_2 G(x_1;x_2) & \cdots & \varepsilon_N\hbar_N G(x_1;x_N) \\ \varepsilon_1\hbar_1 G(x_2;x_1) & \varepsilon_2\hbar_2 G(x_2;x_2)+1 & \cdots & \varepsilon_N\hbar_N G(x_2;x_N) \\ \vdots & \vdots & \ddots & \vdots \\ \varepsilon_1\hbar_1 G(x_N;x_1) & \varepsilon_2\hbar_2 G(x_N;x_2) & \cdots & \varepsilon_N\hbar_N G(x_N;x_N)+1 \end{vmatrix} = 0 \tag{7}$$

Equation (4) with (5) can be solved through the Green's function e.g. the continuous condition and the jump condition at $x = \xi$. The tedious procedure can be eliminated using the construction of Green's theorem which permits explicit expression of the Green's function

$$G(x;\xi) = \frac{1}{(c^2-1)W(\phi_1,\phi_2)\big|_{x=\xi}} \begin{cases} \phi_1(x)\phi_2(\xi),\; 0 \le x \le \xi \\ \phi_1(\xi)\phi_2(x),\; \xi \le x \le 1 \end{cases}. \tag{8}$$

where $W(\phi_1,\phi_2) \ne 0$ holds. Substitution of Eq. (16) into Eq. (7) leads to the frequencies.

The numerical examples show that both the real and the imaginary parts of eigenvalues are varying. It is demonstrated that only the first eigenvalue changes significantly when the eigen-frequency of the oscillator is close to that of the string's first eigenvalue, while all of the eigen-frequencies of the string are significantly influenced by the moving mass model. Here, we briefly show the eigenvalue problem of an axially moving string with two mass-spring oscillators. An equal stiffness of spring $k_1 = k_2 = k$ and different masses m_1 and m_2 are used. The variance of the first three eigenvalues of attached two masses model is depicted in Fig. 1 with $c = 0.25$, $\mu = 0.04$, $m_1 = 0.375$ and $m_2 = 0.3$. Compared with the model of attached oscillator, the effect of moving mass on the first three eigenvalues of the string are significant.

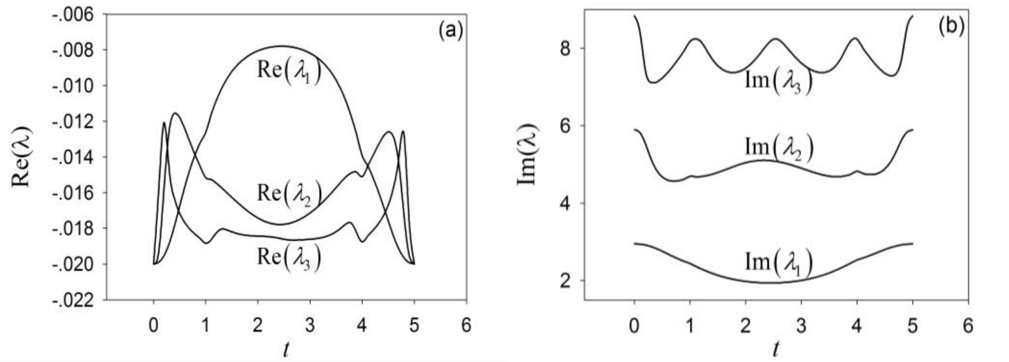

Fig. 1 The first three eigenvalues of the moving masses model: Real parts (left) and imaginary parts (right)

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