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MODE ANALYSIS FOR AN ELASTIC WAVEGUIDE IN A PERIODIC COMPOSITE

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Wave propagation in a periodic structure is characterized by band gaps. Since, in these frequency bands, propagation of any wave modes is forbidden, line defects made in the photonic or phononic crystal can play a role of a waveguide. This paper presents a numerical method for wave modes in such a 2-D elastic waveguide. In order to represent the infinite region and also reduce the computational cost, the equivalent stiffness describing a semi-infinite periodic field is derived in the context of finite element analysis. Considering these conditions, the equation of motion is reduced to a finite element equation for a unit extracted from the waveguide sub-region based on the periodicity along the line defect. The developed approach is applied to waveguides and some fundamental features of the wave modes are examined.

Let us consider an infinite waveguide lying along the x_1 axis in a plane strain periodic composite as illustrated in Fig.1. Since the waveguide region is given by a homogeneous material, the present problem has a periodicity L_1 in x_1 direction. Due to this periodicity, the infinite region can be reduced to a sub-region D_1 of width L_1. Notice that D_1 is still infinite in the vertical direction. The equation of motion of the finite waveguide domain D_0 in D_1 is described as

$$[\hat{\mathbf{K}}]\{\mathbf{U}\} = \{\mathbf{F}\}, \tag{1}$$

where $\{\mathbf{U}\} = \{\mathbf{U}_T\mathbf{U}_B\mathbf{U}_M\mathbf{U}_L\}$, $\{\mathbf{F}\} = \{\mathbf{F}_T\mathbf{F}_B\ \mathbf{0}\ \mathbf{0}\}$. Here $(\ )_T, (\ )_B, (\ )_L$ denote sub-vectors corresponding to the top, bottom and left boundaries of D_0, respectively. $(\ )_M$ is a sub-vector given by the rest of components. Notice that the sub-vectors on the right side has been eliminated by the Floquet's principle as,

$$\{\mathbf{U}_R\} = \mathrm{e}^{-\mathrm{i}\kappa_1 L_1}\{\mathbf{U}_L\}, \quad \{\mathbf{F}_R\} = -\mathrm{e}^{-\mathrm{i}\kappa_1 L_1}\{\mathbf{F}_L\}, \tag{2}$$

where κ_1 stands for the floquet wavenumber in x_1 direction. In eq.(1) the nodal force sub-vectors $\{\mathbf{F}_B\}, \{\mathbf{F}_T\}$ are unknowns. In order to eliminate these values, we introduce equivalent stiffness matrices $[\mathbf{K}_B], [\mathbf{K}_T]$ satisfying the following relations:

$$[\mathbf{K}_B]\{\mathbf{U}_B\} = \{\mathbf{F}_{BD}\}, \quad [\mathbf{K}_T]\{\mathbf{U}_T\} = \{\mathbf{F}_{TU}\}, \tag{3}$$

where $\{\mathbf{F}_{BD}\}$ and $\{\mathbf{F}_{TU}\}$ are nodal forces at the interfaces of the lower and upper semi-infinite periodic regions, respectively. Imposing the equilibrium conditions $\{\mathbf{F}_B + \mathbf{F}_{BD}\} = \mathbf{0}, \{\mathbf{F}_T + \mathbf{F}_{TU}\} = \mathbf{0}$ on the bottom and top boundaries, we can obtain the following equation:

$$\begin{bmatrix} \hat{\mathbf{K}}_{TT} + \mathbf{K}_T & \hat{\mathbf{K}}_{TB} & \hat{\mathbf{K}}_{TM} & \hat{\mathbf{K}}_{TL} \\ \hat{\mathbf{K}}_{BT} & \hat{\mathbf{K}}_{BB} + \mathbf{K}_B & \hat{\mathbf{K}}_{BM} & \hat{\mathbf{K}}_{BL} \\ \hat{\mathbf{K}}_{MT} & \hat{\mathbf{K}}_{MB} & \hat{\mathbf{K}}_{MM} & \hat{\mathbf{K}}_{ML} \\ \hat{\mathbf{K}}_{LT} & \hat{\mathbf{K}}_{LB} & \hat{\mathbf{K}}_{LM} & \hat{\mathbf{K}}_{LL} \end{bmatrix} \begin{Bmatrix} \mathbf{U}_T \\ \mathbf{U}_B \\ \mathbf{U}_M \\ \mathbf{U}_L \end{Bmatrix} = \begin{Bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{Bmatrix}. \tag{4}$$

The guided wave modes are identified as a condition which satisfies eq.(4). The equivalent stiffness matrices $[\mathbf{K}_B], [\mathbf{K}_T]$ are derived through eigenvalue analyses of a unit cell representing the periodic composite[1].

A waveguide shown in Fig.1 is considered. The density and stiffness ratios of the circular inclusion to the base material are ρ_b/ρ_a=9.6, and μ_b/μ_a=3.5. The Poisson's ratios are ν_a=0.37 and ν_b=0.44. The relationship between the width of the waveguide h and the normalized mode frequency $\omega L_1/C_T$ is shown in Fig.2 for κ_1=0, here C_T is speed of the transvers wave in the base material. From the figure, it can be seen that each mode strongly depends on the width h. These modes can be classified into two waves, that is, horizontally and vertically polarized modes.

REFERENCES

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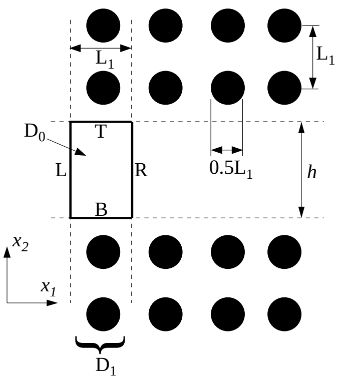

Fig.1 Waveguide in a periodic field.

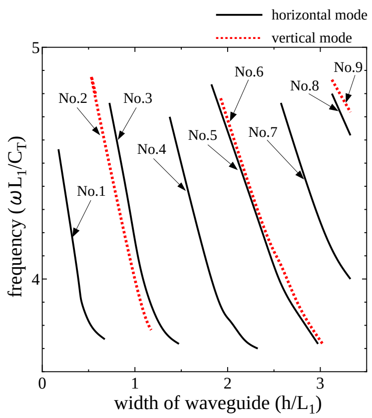

Fig.2 Influence of waveguide width on mode frequencies ($\kappa_1 = 0$).