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MOMENTUM INTERPOLATION FOR QUASI ONE-DIMENSIONAL UNSTEADY LOW MACH NUMBER FLOWS WITH ACOUSTICS

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When a co-located arrangement of the unknowns is adopted, compressible low Mach number flow calculations may encounter difficulties heavily related to the way of interpolation on the cell or element faces (see *e.g.* [1, 2, 3, 4, 5, 8]). In particular, loss of accuracy may arise for unsteady calculations, when convective and acoustic waves propagate together at very different time and space scales, with possible interactions.

Versions of Godunov-type schemes that remain accurate at low Mach number are designed such as to conform to some low Mach number continuous asymptotic properties (see *e.g.* [1, 2, 3, 4]). One of these properties is that the thermodynamic and acoustic pressures should be constant in space at the convective scale [7]. Denoting by $\mathrm{M_r}$ a reference Mach number in the flow, it was shown in [4] for steady flow calculations, that this requests the $1/\mathrm{M_r^2}-$scaling of the pressure gradient term in the face velocity or the face mass flux to be preserved. If suitable boundary conditions are chosen, the checkerboard decoupling problem that may arise at low Mach number is thus avoided. As shown in [1, 3], another asymptotic property provides insights for the design of Godunov-type schemes that remain accurate at low Mach number. This property is the linear acoustic energy conservation in the low Mach number regime, which holds if periodic boundary conditions are adopted. Assessment of the growth rate of the discrete linear acoustic energy (due to spurious acoustic waves) was thus used in [1] to design a modified Godunov-type scheme accurate at any Mach number for the compressible Euler system. The guideline is the study of the behaviour at low Mach number of the first-order modified equation associated to the Godunov scheme applied to the linear wave equation. In [2], this approach is also used in the case of the quasi one-dimensional linear acoustic equation.

In the present study, we propose to justify in the low Mach number regime the momentum interpolation technique, also often called Rhie-Chow interpolation technique [10], applied to the quasi one-dimensionnal compressible Euler equations. Notice that from the momentum equation, the above mentioned $1/\mathrm{M}_\mathrm{r}^2-$scaling of the pressure gradient term in the face velocity is readily satisfied. As a consequence, the momentum interpolation method does not suffer from checkerboard modes. The ability of the momentum interpolation method to properly simulate acoustic waves in low Mach number flows is justified by expliciting the first-order modified equation associated to this scheme in the case of the quasi one-dimensional linear acoustic equation. It is shown that, at the discrete level, the linear acoustic energy behaviour is similar to that at the continuous level. It is expected that this will be beneficial for accurate calculation of low Mach number flows in the non-linear case, including acoustics, in nozzles with variable cross-section area for which some numerical results are presented.

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