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A CELL-CENTERED PRESSURE CORRECTION SCHEME FOR THE COMPRESSIBLE EULER EQUATIONS

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Defining a robust scheme for the numerical solution of the compressible Euler equations for all Mach regimes is a challenging issue. Indeed, in the zero Mach limit, the pressure gradient has a singular limit and the acoustic time scale vanishes. Consequently, explicit schemes, often based on (approximate) Riemann solvers, face severe limitations, among which the loss of accuracy of the pressure gradient approximation and the time step limitation. On the other hand, pressure-correction methods may be relevant for addressing this issue, in particular because of their built-in stability properties.

In this work, we present an original scheme falling in this class of methods, namely a cell-centered pressure-correction scheme extending the ideas developed in [1, 2] for staggered space discretizations. We use a two-step algorithm: first, the momentum balance is solved for a tentative velocity, then this latter is corrected in a step which couples the mass and the energy balance equations. Upwinding is performed in a decoupled manner (*i.e.* equation-per-equation), and with respect to the material velocity. In the prediction step, the pressure gradient is built on the basis of the transpose of the divergence operator (and thus centered in space), with an *ad hoc* corrective factor which allows to derive a discrete kinetic energy balance. This latter relation includes remainder terms, which correspond to the dissipation introduced by the numerical viscosity and do not tend to zero (in a distribution sense) at the limit of vanishing time and space steps. Subtracting it to an approximation of the total energy balance, we obtain a discrete internal energy balance, in which source terms compensating the above mentioned residual terms appear. This is this form of the energy equation, which ensures the positivity of the internal energy, which is used in the correction step.

Thanks to the upwind approximation of the mass balance, the density is also positive and therefore the algorithm keeps the solution in the convex of admissible states. In addition, it is shown to preserve the energy of the flow (*i.e.* the integral of the total energy over the computational domain), and to keep the velocity and pressure constant across the 1-dimensional contact discontinuity. Finally, we prove its consistency, in the sense that a limit of a converging sequence of solutions is shown to satisfy the weak form of the Euler equations.

In our numerical experiments, several 1D and 2D Riemann problems are tested. Our scheme shows a good accuracy near contact-waves and rarefactions. Shocks are sharply computed though oscillations are observed for strong shocks. These oscillations are damped using the WLR adaptive artificial viscosity method [3] without affecting the first order accuracy of the scheme.

REFERENCES

[1] D. Grapsas, W. Kheriji, R. Herbin and J.-C. Latché. An unconditionally stable Finite-Element-Finite Volume pressure correction scheme for the compressible Navier-Stokes equations. *(in preparation)*, 2013.

[2] R. Herbin, W. Kheriji and J.-C. Latché. Consistent pressure correction staggered schemes for the shallow water and Euler equations. *M2AN (submitted)*, 2013.

[3] A. Kurganov and Y. Liu. New adaptive artificial viscosity method for hyperbolic systems of conservation laws. *Journal of Computational Physics*, Vol. **231**, 8114–8132, 2012.

[4] P.D. Lax and X.-D. Liu. Solution of two-dimensional Riemann problems of gas dynamics by positive schemes. *SIAM Journal on Scientific Computing*, Vol. **19**, 319–340, 1998.

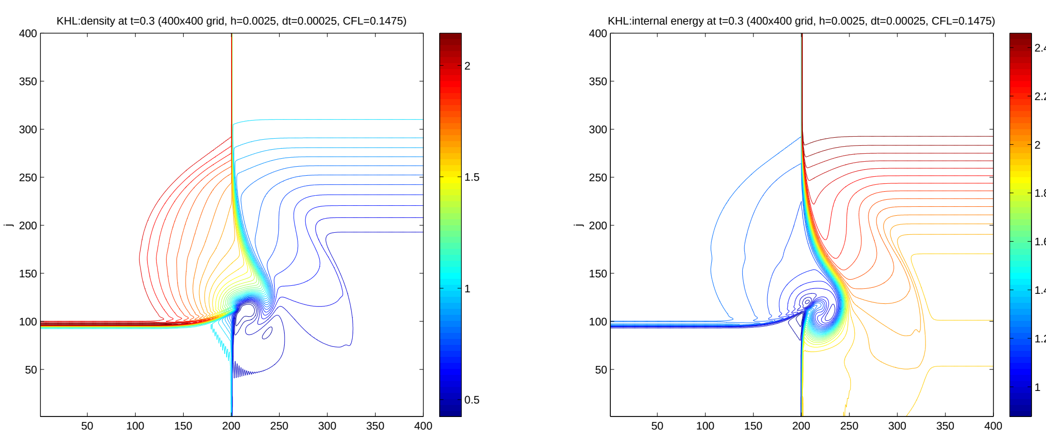

Figure 1: Riemann problem from configuration 17 in [4] (40 contours).