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COMPARISION OF ALTERNATIVE APPROACHES FOR THE STOCHASTIC FINITE ELEMENT ANALYSIS OF STRUCTURES WITH ELASTO-PLASTIC DAMAGE BEHAVIOR

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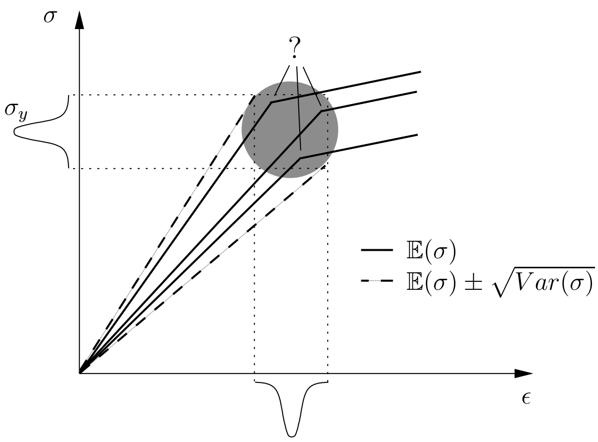
Uncertainties in all of the system parameters like geometry, boundary conditions or material properties can be considered. Various methods have been developed to deal with these kinds of uncertainties. In this work random material parameter and deterministic boundary conditions are investigated. There exists a broad range of techniques to treat these uncertainties in the framework of Finite Element Method often called *Stochastic Finite Element Method*.

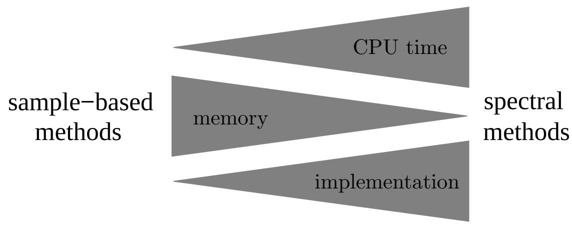
The well-kown *Monte Carlo Method* is the first method to be mentioned, which consists of calculating the underlying deterministic system n-times with varying parameters. The stochastic system response can be calculated by the statistics of these samples. This method, like every other sample-based technique, works with every kind of deterministic system. An improvement of the Monte Carlo Method with its relatively bad convergence behavior of $1/\sqrt{n}$, is besides many other the *Latin Hypercube Sampling*, in which the samples are generated on a grid within the stochastic space.

Different methods as well based on sampling strategies are the *Collocation Methods*. The underlying problem is calculated at sample points, which build a grid in the stochastic space. The stochastic solution is an interpolation of the samples by suitable polynomials. The *Smolyak Algorithm* [3] is based on a Chebychev grid and uses Lagrangian polynomails.

Another approach to deal with uncertainties are the non-sample based or non-intrusive methods. *The Polynomial Chaos* [2] approximates the distribution of a random variable by a series expansion of deterministic coefficients and polynomials which arguments are random variables with a suitable probabilistic measurement. This method can be incorporated into a Finite Element code, but several changes especially for the mathematical operations likewise multiplication and division, has to be made. In comparision to the



(a) stress-strain curve for plasticity with linear isotropic hardening with uncertain stress and strain measurements



(b) different characteristics of sample-based and non-sample based methods for the calculation of random systems

sample-based methods one has to calculate the underlying problem just once. But the dimension of the problem is much higher than in the deterministic case.

Every mentioned method has its own advantages and disadvantages. The main characteristics that has to be taken into account are computation time, computational power (cpu and user memory) and the implementation efford which goes hand in hand with the process of getting a deeper knowledge of the methods. One can see for the two classes of methods that they complement each other (see figure (b)). Each technique results in a probability density function for the system response of interest like stresses (cp. first and second order methods), that is why these three characteristics are important to choose between them.

For a comparision of the three methods a Finite Element framework is set up. All three methods are applied to a 3D mesh with suitable boundary conditions and some stochastic material parameters. For the material description the non-linear elasto-plastic damage model after Lemaitre [1] has been chosen.

Because the values for the evaluated yield criterion are no longer scalar ones but a probability represented by its Polynomial Chaos coefficients the decision if plasticity occurs needs special treatment. This problem is depicted in figure (a). For the solution of this problem the most important probability value, the expectation, is used to set up a suitable criterion for the start of the plasticity calculation.

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