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Rational selection of experimental data for inverse structural problems

C. Chisari[1]*, L.Macorini[2], C. Amadio[3] and B.A. Izzuddin[4]

[1] University of Trieste, Piazzale Europa, 1, 34127, Trieste, Italy, corrado.chisari@gmail.com
[2] Imperial College London, South Kensington Campus, London SW7 2AZ, United Kingdom, l.macorini@imperial.ac.uk
[3] University of Trieste, Piazzale Europa, 1, 34127, Trieste, Italy, amadio@units.it
[4] Imperial College London, South Kensington Campus, London SW7 2AZ, United Kingdom, b.izzuddin@imperial.ac.uk

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Inverse analysis has been established as an effective and sound tool for the calibration of model parameters in a wide range of engineering problems. Among others, it has been successfully applied in fluid dynamics, electrical problems, structure dynamics, solid mechanics, soil-structure interaction, detection of inclusions and cracks, fracture mechanics. In recent years, several methods for solving inverse problems have been proposed [1], including i) minimization of the discrepancy between measured and computed data, ii) constitutive gap method, iii) virtual fields method, iv) equilibrium gap method and v) reciprocity gap method. When applied to structural problems, all these methods, except for the first approach, require a complete set of measured data (i.e. full-field measurements) which can be obtained using optical devices (Digital Image Correlation, Moiré and Speckle interferometry, Grid methods) and are usually applied to simple experimental setups on small specimens. However, as very often only a limited set of measured data is available, the minimization of the discrepancy function represents the most popular technique used to solve inverse structural problems. This involves the use of an optimization algorithm to find the input vector (i.e. the vector of the sought parameters) which determines the minimum discrepancy between measured and computed data. While this is zero only in ideal experiments with no errors, in real cases measurement and modelling errors corresponding to noise effects in the optimisation procedure are always present and should be properly taken into account.

Inverse problems very often suffer from ill-conditioning. According to Hadamard's definition [2], an inverse problem is well-posed when i) the solution exists, ii) it is unique, and iii) it is stable, i.e. when a small noise is applied to the known term, the solution of the "perturbed" problem remains in the neighbourhood of the "exact" solution. Moreover, even if the existence and the uniqueness conditions hold, it is important to study the stability of the solution accounting for noise effects, since, as mentioned before, they cannot be neglected in real problems.

An inverse problem may be ill-conditioned in a global sense or only for specific measurement data. In the first case, it is impossible to determine the input vector by means of inverse techniques even if the full-field measurements are known. This means that the experimental setup has been poorly chosen, and the sensitivity of the response (in a global sense) to the variation of the input data is very low or null. In the latter case, even if the problem is globally well-posed, the sought parameters cannot be obtained using the chosen experimental data or the results could be strongly influenced by noise effects. Thus, since type, number and location of the sensors used in the experimental tests are usually chosen in an empirical way, a rational methodology for the assessment of the experimental equipment is critical to exploit the full potential of the inverse procedure.

In this work, an enhanced methodology for a structural inverse problem is presented, where the focus is on the determination of material parameters for a detailed mesoscale model for unreinforced masonry [3]. This was investigated by the authors in previous research [4], where the need for an optimal set of experimental data was pointed out. The proposed methodology utilises Proper Orthogonal Decomposition [5] as a valuable tool to assess the representativeness of a limited set of measured data to capture the global response of the structure. Known also as Principal Component Analysis, Karhunen-Loeve Decomposition, Singular Value Decomposition, it allows for the determination of a reduced basis of linearly uncorrelated variables called principal components. Thus the global response is approximated as a linear combination of some modal shapes, where the principal components (i.e. the amplitudes) can be considered as a "reduced" but accurate representation of the global response. This work shows the optimisation of metrology (number, quality and location of sensors) to minimize the effect of the noise data on the amplitudes, which, in the end, corresponds to minimising the effect of the noise data on the global response description. It is also shown how this reflects on the stability of the inverse problem. Finally, it is important to point out that although this work focuses on a specific structural static problem, the proposed methodology is easily extendable to more general applications.

REFERENCES

[1] S. Avril et al., Overview of Identification Methods of Mechanical Parameters Based on Full-field Measurements. *Experimental Mechanics*, Vol. **48**, pp. 381-402, 2008.

[2] S.I. Kabanikhin, Definitions and examples of inverse and ill-posed problems. *J. Inv. Ill-Posed Problems*, Vol. **16**, pp. 317-357, 2008.

[3] L. Macorini, B.A. Izzudin, A non-linear interface element for 3D mesoscale analysis of brick-masonry structures, *Int. J. Numer. Meth. Engrg.*, 85: 1584-1608, 2011.

[4] C. Chisari, L. Macorini, C. Amadio, B.A. Izzuddin, Identification of Brick-Masonry Material Properties Through Inverse Analysis and Genetic Algorithms, in B.H.V. Topping, P. Iványi, (Editors), *Proceedings of the Fourteenth International Conference on Civil, Structural and Environmental Engineering Computing*, Civil-Comp Press, Stirlingshire, UK, Paper 70, 2013. doi:10.4203/ccp.102.70

[5] Y.C. Liang, H.P. Lee, S.P. Lim, K.H. Lee, W.Z. Lin, Proper Orthogonal Decomposition and its applications — part i: theory. *J Sound Vibration*, Vol. **252(3)**, 527–544, 2002.