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FAST HIERARCHICAL SOLVERS FOR DISCONTINUOUS GALERKIN METHODS

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We present an iterative method for solution of linear systems resulting from discontinuous Galerkin (DG) approximations. The two-level solution strategy is based on hierarchical scale separation (HSS) such that the linear system is solved globally only for the mean values of the unknowns (coarse scales). The system matrix of this coarse scale problem is exactly the same as in the cell-centered finite volume method. The higher order components of the solution (fine scales) are computed as corrections by solving small local problems. This technique is particularly efficient for DG schemes that use hierarchical bases and leads to an unconditionally stable method for stationary and time-dependent hyperbolic and parabolic problems. Unlike p-multigrid schemes, only two levels are used for DG approximations of any order. The proposed method is conceptually simple and easy to implement. It compared favorably to p-multigrid in our numerical experiments. Numerical tests confirm the accuracy and robustness of the proposed algorithm.

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