

EXACT FORMULAS FOR BENDING OF SANDWICH BEAMS USING THE REFINED ZIGZAG THEORY

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Key Words: *Refined Zigzag Theory, Sandwich Beams, Analytic Formulas, Three-Point, Bending, Four-Point Bending.*

The Refined Zigzag Theory (RZT) is a recently developed model for the analysis of multilayered composite and sandwich beams and plates [1-3]. The theory is an improvement of the First-order Shear Deformation Theory (FSDT) since it takes into account the normal distortion (*zigzag effect*), typical of multilayered structures, through the enrichment of the FSDT in-plane displacement field with *zigzag* additional terms. The through-the-thickness shape of these terms is described by the so-called *zigzag functions* (piecewise linear and depending on the thickness and on the shear stiffness of each layer) whereas the amplitude of the normal distortion is measured by the *zigzag rotations*, two kinematic variables that are additional with respect to the classical five of FSDT.

Several papers have already been published demonstrating the accuracy and computational efficiency of RZT-based analyses. Analytic approaches (of the Navier's type or with the Rayleigh-Ritz method [1-5]) and FEM solutions (beam [6,7] and plate [8,9]) have been used to evaluate the static response, the free vibration modes and the buckling loads of multilayered composite and sandwich structures. Nevertheless, a more practical engineering approach for the solution of the same problems would require simple formulas based on the key material and geometry properties of the structure.

The present work aims at developing, in the framework of RZT, exact-analytic formulas for the static response of beams in planar bending. The focus will be on sandwich-like stacking sequences and on typical boundary and loading conditions. This study will also be the occasion to investigate how the normal distortion (*zigzag*) affects the maximum deflection of sandwich beams and how the shear correction factor for Timoshenko beam theory can be calculated in the framework of RZT.

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