11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
July 20–25, 2014, Barcelona, Spain

NATURAL VIBRATIONS AND STABILITY OF NON-CIRCULAR FGM SHELLS CONTAINING FLUID

Sergey A. Bochkarev, Sergey V. Lekomtsev* and Valery P. Matveenko

Institute of Continuous Media Mechanics UB RAS, Acad. Korolev Str. 1, Perm 614013, Russia
bochkarev@icmm.ru, lekomtsev@icmm.ru, mvp@icmm.ru

Key words: *stability, natural vibrations, elliptical cylindrical shells, FEM, FGM.*

A variety of technical applications, which use functionally graded materials (FGM) generates a need for stability analysis of structures made of these materials. Generally, these investigations have focused on natural vibrations and stability of empty shells and only a small number of resaerch papers are devoted to functionally graded shells containing fluid. To our knowledge there is only one paper in the literature [1], which analyzes the shells filled with incompressible fluid. However, fluid conveying structures with slightly non-circular or non-circular cross-sections are extensively used in modern technologies. This is a strong reason for the development of advanced numerical algorithms allowing us to simulate their behavior. This paper presents a 3D formulation of the spectral problem and finite element algorithm (FEM) for its numerical implementation, designed to investigate natural vibrations and stability of prestressed functionally graded shells with arbitrary cross-section interacting with a quiescent or flowing compressible fluid.

The motion of non-viscous fluid is described by the wave equation, which together with the impermeability condition and appropriate boundary conditions are transformed using the Bubnov–Galerkin method. Simulation of the shell with arbitrary cross-section is carried out on the assumption that its curvilinear surface is approximated to sufficient accuracy by a set of plane rectangular elements [2]. The strains are calculated using the relations of the Kirchhoff–Love's thin shells theory. A mathematical formulation of the dynamic shell problem has been developed by applying the variational principle of virtual displacements

$$\int_{S_s} \delta\boldsymbol{\varepsilon}^{\mathrm{T}}\mathbf{D}\boldsymbol{\varepsilon}dS + \int_{V_s} \rho_s \delta\boldsymbol{d}^{\mathrm{T}}\ddot{\boldsymbol{d}}dV - \int_{S_\sigma} \delta\boldsymbol{d}^{\mathrm{T}}\boldsymbol{P}dS + \int_{S_s} \delta\boldsymbol{e}^{\mathrm{T}}\boldsymbol{\sigma}_0\boldsymbol{e}dS = 0.$$

Here: $\boldsymbol{e} = \left\{\frac{\partial u}{\partial x}, \frac{\partial v}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial w}{\partial x}, \frac{\partial w}{\partial y}, \frac{\partial^2 w}{\partial x^2}, \frac{\partial^2 w}{\partial y^2}, \frac{\partial^2 w}{\partial x \partial y}\right\}^{\mathrm{T}}$; $\boldsymbol{\varepsilon} = \left\{\frac{\partial u}{\partial x}, \frac{\partial v}{\partial y}, \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}, \frac{\partial^2 w}{\partial x^2}, \frac{\partial^2 w}{\partial y^2}, 2\frac{\partial^2 w}{\partial x \partial y}\right\}^{\mathrm{T}}$; $\boldsymbol{P} = \{0, 0, -\rho_f\left(\partial\phi/\partial t\right), 0, 0, 0\}^{\mathrm{T}}$; u, v and w are components of the displacement vector of the shell; V_s and S_s are the shell domain and lateral shell surface; $\boldsymbol{d}$ and $\boldsymbol{P}$ are the vectors of generalized displacements and hydrodynamic load on the surface S_σ; ρ_s and

ρ_f are the densities of the shell material and fluid, respectively; $\mathbf{D}$ is the matrix of physical constants of FGM in which the effective properties are evaluated according to the power law; (x, y, z) are the coordinates associated with the lateral surface of the shell; ϕ is perturbation velocity potential. The entries of the matrix $\boldsymbol{\sigma}_0$ are determined from the condition $\mathbf{E}^{\mathrm{T}}\mathbf{D}\boldsymbol{\varepsilon}_0 = \boldsymbol{\sigma}_0\boldsymbol{e}$. Here $\mathbf{E}$ is the matrix of linear multipliers, and vector $\boldsymbol{\varepsilon}_0$ is found by solving the corresponding static problem $\mathbf{K}_s\boldsymbol{d} = \boldsymbol{P}_0$, where $\boldsymbol{P}_0$ is the vector of static mechanical and thermal loads.

Using FEM we reduce the problem of the dynamic behavior of prestressed FGM shells containing quiescent or flowing fluid to a coupled system of two equations which can be transformed to a standard eigenvalue problem

$$\begin{bmatrix} 0 & \mathbf{I} \\ -\mathbf{M}^{-1}(\mathbf{K}+\mathbf{A}) & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix} \boldsymbol{x} = \omega \boldsymbol{x}, \quad \boldsymbol{x} = \left\{ \boldsymbol{q}, \boldsymbol{f}, \dot{\boldsymbol{q}}, \dot{\boldsymbol{f}} \right\}^{\mathrm{T}}. \tag{1}$$

Here $\mathbf{K} = \mathrm{diag}\{\mathbf{K}_s + \mathbf{K}_g, \mathbf{K}_f\}$ is the stiffness matrix, $\mathbf{M} = \mathrm{diag}\{\mathbf{M}_s, \mathbf{M}_f\}$ is the mass matrix, $\mathbf{C} = \begin{bmatrix} 0 & \rho_f\mathbf{C}_s \\ -\mathbf{C}_f & -\mathbf{C}_f^c \end{bmatrix}$ is the hydrodynamic damping matrix, $\mathbf{A} = \begin{bmatrix} 0 & \rho_f\mathbf{A}_s \\ -\mathbf{A}_f & \mathbf{A}_f^c \end{bmatrix}$ is the hydrodynamic stiffness matrix, $\mathbf{K}_g = \int_{S_s} \mathbf{G}^{\mathrm{T}}\boldsymbol{\sigma}_0\mathbf{G}dS$ is the matrix of the geometrical stiffness, $\mathbf{I}$ is the unit matrix, $\boldsymbol{q}$ and $\boldsymbol{f}$ are some functions of coordinates, $\mathbf{G}$ is the matrix relating the strains $\boldsymbol{e}$ to the nodal displacements, ω is the complex characteristic quantity. A more detailed description of the submatrices contained in (1) is presented in [3].

Several numerical examples have been considered to analyze the influence of linear dimensions, fluid levels and boundary conditions on the natural frequencies, vibration modes and hydrodynamic stability boundary of thin-walled functionally graded circular and elliptical cylindrical shells interacting with a quiescent or flowing fluid and subjected to static mechanical and thermal loads. The effects of hydrostatic pressure and heating of the lateral surface of the functionally graded shells on their dynamic characteristics are evaluated. It is shown that the mechanical and hydroelastic stability boundary of such structures can be controlled through the variation of the volume fraction of each constituent material.

This study is supported by the Russian Foundation for Basic Research (grants 12-01-00323 and 13-01-96049).

REFERENCES

[1] Z. Iqbal, M.N. Naeem, N. Sultana, S.H. Arshad and A.G. Shah. Vibration characteristics of FGM circular cylindrical shells filled with fluid using wave propagation approach. *Appl. Math. Mech.*, Vol. **30**, 1393–1404, 2009.

[2] O.C. Zienkiewicz and R.L. Taylor. *The Finite Element Method*, Elsevier, 2005.

[3] S.A. Bochkarev. S.V. Lekomtsev and V.P. Matveenko. Natural vibrations and stability of shells with arbitrary cross-section, containing flowing fluid. *Proc. 11th International Conference on Vibration Problems, Lisbon, Portugal*, 2013.