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ON CONVECTIVE STRUCTURES IN VERTICAL CYLINDER UNDER RADIATIVE HEATING

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Investigations of the physicochemical and biological characteristics of “age–specific” composition of water at Lake Baikal indicate the presence of deep mixing mechanism that transfers surface water to bottom regions [1, 2]. The purpose of the study is to suggest the mathematical model describing the appearance and behavior of the convective structures which arise in a water layer of natural basin. For this the exact solution with the free parameter of stationary Navier–Stokes equations is investigated in detail. The solution has the following form

$$\begin{aligned} &\mathbf{u} = (u_1, v_1, w_1), \quad u_1 = u(z)r, \quad v_1 = 0, \quad w_1 = w(z), \\ &\theta = \theta(z), \quad p = p_1(z) + a_1 r^2/2, \end{aligned} \tag{1}$$

where u_1, v_1, w_1 are the radial, azimuthal and axial velocity components respectively, p is the pressure, a_1 parameter is to be defined. The solution is the axisymmetrical analogue of Hiemenz solution of Navier–Stokes equations in a plane case. It is found that the solution of the equations of thermal gravitational convection in the Oberbeck–Boussinesq approximation taking into account the radiative heating and non–monotonic dependence of liquid density on temperature which is typical to water near 4^oC can be obtained with help of solution (1). It is connected with recurrent nature of the system of the basic equations. In the dimensionless variables the governing equations in cylindrical coordinates have form

$$\begin{aligned} &2u + w_z = 0, \quad u^2 + wu_z = u_{zz} + a, \\ &ww_z = -p_z + w_{zz} + \mathrm{Ga} - \mathrm{Gr}(\theta - 1)^2, \\ &\theta_z w = \frac{1}{\mathrm{Pr}}\,\theta_{zz} + f_1 F_1(z), \end{aligned}$$

Here a is the non–dimensional independent parameter, Ga, Gr, Pr are the Galileo, Grashof and Prandtl numbers respectively, f_1 is the heat release parameter, $F_1(z)$ is the heat source function describing the absorption of solar radiation.

The original problem is divided into consequently solved problems for u, w, p, θ. The basic one of them is the non–linear two–point problem for u function

$$u_{zz} + 2u_z \int_0^z u(z)dz - u^2 + a = 0, \quad 0 < z < 1,$$
$$u(0) = u(1) = 0, \quad \int_0^1 u(z)dz = 0.$$

The last problem reduces to the operator equation in $C[0,1]$ space and existence of its solution is clarified on base of Schauder's theorem. As it is well known Schauder principle does not guarantee the solution uniqueness. Numerical analysis of the problem finds the presence of three fixed points of the appropriate image and corresponding values of a parameter. The iterative procedure for finding the free parameter is suggested.

The linear stability of all classes of solutions is studied numerically. Critical thermal mode is isolated. Evolution of oscillatory mode depending on Prandtl number is investigated. It is shown that under small Prandtl numbers oscillatory modes decay. If Prandtl numbers are not small the instability is connected with growing thermal disturbances. Another instability mechanism is found out in the short waves domain. In this case the crisis attributes to growing hydrodynamical disturbances.

The small parameter method is used for the determining of supercritical regimes of the flow in the functions class which are periodical with respect to r coordinates and which are branched out from the basic flow (1). Performed calculations show in the entire domain of existence plane mode the secondary regimes exist for all values of a parameter. The flow is the axisymmetrical and has the complex many–vortex structure.

The method of finite differences is used in the domain of supercritical values of the Grashof number for the oscillatory instability. It is found out that several vortices performs periodic oscillations near the inversion coordinate. There is certain time–lag with that vortices oscillations respond to oscillations of thermal field.

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