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A SPACE-TIME CUT FINITE ELEMENT METHOD FOR CONVECTION-DIFFUSION PROBLEMS ON TIME DEPENDENT SURFACES

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There is a growing interest in accurate and efficient numerical methods for solving partial differential equations (PDEs) on surfaces. We consider a time dependent convection diffusion equation governing the evolution of the surfactant concentration on a moving interface that separates two immiscible fluids [1, 2]. Surfactants are important because of their ability to reduce the surface tension and are for example used in detergents, oil recovery, and in treatment of lung diseases.

PDEs on a given surface can be accurately solved if the mesh is aligned with the surface. However, the requirement that the mesh should conform to the interface leads to significant complications for two phase flow simulations, where the interface evolves with time and may undergo topological changes. Therefore it would be preferable to use a method in which the interface can be arbitrarily located with respect to a fixed background mesh.

Recently, a finite element method for the Laplace–Beltrami operator where the surface is allowed to cut through the background grid in an arbitrary fashion was proposed in [3]. We call this type of methods **cut finite element methods** since the surface can cut through the background grid. The cut finite element spaces are constructed by first embedding the surface in a background grid equipped with a standard finite element space and then taking the restriction of these functions to the surface. A drawback of this type of methods is that the stiffness matrix may become arbitrarily ill conditioned depending on the position of the surface in the background mesh. In the case of the Laplace–Beltrami operator this ill conditioning has been addressed in [4] and [5].

We present a cut finite element method for a time dependent convection diffusion equation on an evolving surface governing the evolution of the surfactant concentration. In [6] we presented a first order accurate finite element method based on a characteristic-Galerkin formulation combined with a piecewise linear cut finite element method in space. We now present an optimal order accurate cut finite element method based on a space-time approach with continuous linear elements in space and discontinuous linear elements in time. To ensure well posedness of the resulting algebraic systems of equations, independent of the position of the interface in the background grid, we add a consistent stabilization term. Numerical results in both two and three dimensions indicate that the method is optimal order accurate and we have analytical results for the convection dominated case. In addition, the total mass of surfactant can be accurately conserved using a Lagrange multiplier. The proposed finite element method is very convenient for two phase flow simulations since the same finite element spaces can be used for solving problems both in the bulk domain and on the interfaces separating immiscible fluids. Our method works both with explicit and implicit interface representation techniques.

REFERENCES

[1] H. A. Stone, A simple derivation of the time-dependent convective-diffusion equation for surfactant transport along a deforming interface, *Physics of Fluids A: Fluid Dynamics* Vol. **2** (1), 111–112, 1990.

[2] H. Wong, D. Rumschitzki, C. Maldarelli, On the surfactant mass balance at a deforming fluid interface, *Physics of Fluids* Vol. **8** (11), 3203–3204, 1996.

[3] M. A. Olshanskii, A. Reusken, J. Grande, A finite element method for elliptic equations on surfaces, *SIAM J. Numer. Anal.* Vol. **47**, 3339 – 3358, 2009.

[4] M. A. Olshanskii, A. Reusken, A finite element method for surface PDEs: matrix properties, *Numer. Math.* Vol. **114** (3), 491–520, 2010 .

[5] E. Burman, P. Hansbo, M. G. Larson, A Stable Cut Finite Element Method for Partial Differential Equations on Surfaces: The Laplace-Beltrami Operator, *arXiv:1312.1097*, 2013.

[6] Peter Hansbo, Mats G. Larson, Sara Zahedi, Characteristic Cut Finite Element Methods for Convection-Diffusion Problems on Time Dependent Surfaces, *Technical report 2013-004, Uppsala University*, 2013.