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MULTISCALE GALERKIN METHODS FOR THE EFFICIENT NUMERICAL SIMULATION OF WAVE PROPAGATION IN HETEROGENEOUS MATERIALS WITH REPEATED PATTERNS

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The advanced control of wave propagation is a very desirable ability. It allows for the design of advanced materials in acoustics, elasticity and electromagnetics such as cloaks, energy harvesters, photonic/phononic crystals... To that end, the accurate numerical simulation of wave problems is crucial and thus it has recently been a very active field of research.

In this presentation we introduce a multiscale continuous Galerkin (MCG) and a multiscale discontinuous Galerkin (MDG) methods for Helmholtz's and Maxwell's equations in 2d and 3d. We then apply the methods to solve problems arising from Photonic Crystals to acoustic metamaterials. These methodologies have already been used for other applications such as compressible flows [2]; however they work particularly well for problems of this kind since repeated patterns are found very often. The local subproblems of equivalent patterns are only solved once and reused when necessary. This procedure improves substantially the numerical efficiency with respect to other Finite Element method based approaches.

Firstly, the governing equation on a bounded polygonal/polyhedral domain is decomposed into a set of local subproblems, which are later resolved (only one representative of each equivalence class) either through a CG or DG method. The local solutions are then assembled into a global system for the traces on the edges/faces that needs to be solved. Finally, the local solutions are reconstructed from the traces of each subelement as collection of totally parallelizable Dirichlet problems. Numerical results for Photonic crystal waveguides, nanocavities and superlenses are presented to demonstrate the accuracy and performance of these methods.

Figure 1 shows the numerical solution of a three dimensional waveguide on a Photonic Crystal slab obtained using this method. While the total number of degrees of freedom is about 10 milion, the multiscale reduced global problem on the traces has only 500k degrees of freedom and therefore, this solution has been obtained in just a few minutes (without extremely powerful computational resources) thanks to such reduction of the system size. The high efficiency and accuracy of these methods allow for a posterior design optimization of structured materials, as introduced in [1] and also applied to the design of cloaks and high transmission waveguide bends.

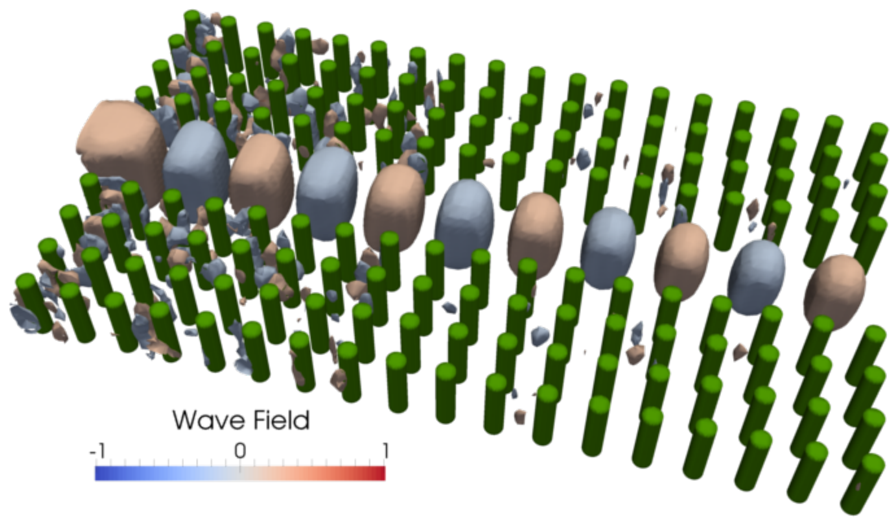

Figure 1: 3d Photonic Crystal slab waveguide

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