11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
July 20–25, 2014, Barcelona, Spain

INTRODUCING A FINITE DIFFERENCE ELEMENT METHOD FOR THIN ELASTIC SHELL

Daniel Choï[1]

[1] LMNO – Université de Caen Basse-Normandie, Bld Maréchal Juin, 14032 Caen Cedex, France

Key words: *Thin elastic shells, membrane locking, finite element method*

The Finite Element Method based on polynomial interpolation is widely and successfully used in engineering applications. Among major problems encountered in Finite Element Methods is the numerical locking. Classical examples are incompressible continuum, shear locking in beam and plates element and shear and membrane locking in thin shells computation. The main difficulty in shells computations is the extreme variety of asymptotic behaviors depending on the geometry, boundary conditions and external loading : a perfect shell element must be *locking free* for bending dominated behaviors and in the same time be effective in membrane dominated and mixed behaviors [4].

Various finite element methods are claimed successful with regard to the locking problems [1, 3, 2, 5, 8], however up to our knowledge, no mathematical proof is available yet. In [7], the shape functions has been identified as closely related to numerical locking for conformal finite elements. For that reason, we wanted to explore the possibility to implement a finite element procedure without shape functions as an alternative. Based on this idea, we introduce a *Finite Difference Element Method* concept and we present a very simple four nodes quadrangular Finite Difference Element procedure (FDEM4) for thin elastic shell problems, based on Naghdi's model.

The basic idea of FDEM4 is quite simple : In classical finite element method, the computation of the so called 'rigidity' matrix need the explicit expression of the shape functions and their derivatives on the nodes of a chosen integration scheme. Here, the Finite Difference Element Method consists simply in integrating over nodes where we replace differential expressions by some consistent finite difference approximations. In FDEM4, we simply choose a single node integration where the derivatives are obtained through standard finite difference schemes. In short, the shape functions are no longer needed, an extension of the method can even work with incompatible meshes.

We obtain then a very simple element to implement. In the case of thin elastic shells, using Nagdhi's model, we obtain very promising numerical results for usual tests and with respect to the membrane locking, compared to some widely used element such as MITC4 or DKT.

As an example, we show in Figure 1 representing the behaviors of MITC4, DKT and FDEM4 elements for a partially clamped non-inhibited hyperbolic mid surface shell for relative thickness of order 0.01 and 0.001. The reference solution is the solution of the bending limit problem, see [6].

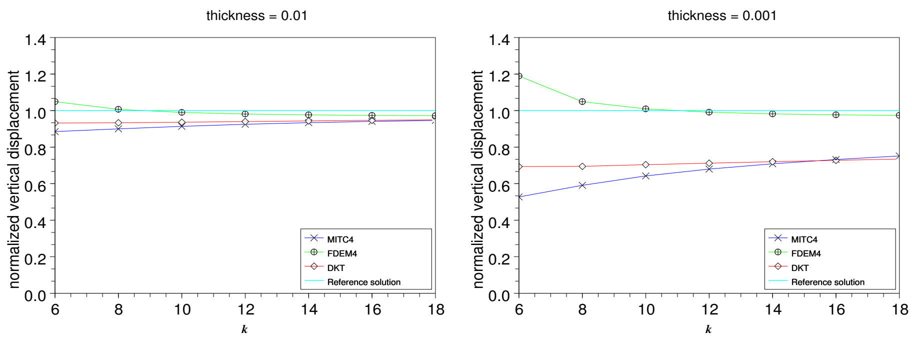

Figure 1: Non-inhibited partially clamped hyperbolic paraboloid. Normalized vertical displacement on $k \times k$ distorted meshes

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